Need vs. Merit: The Large Core of College Admissions Markets

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Abstract

We study college admissions markets, where colleges offer multiple funding levels. Colleges wish to recruit the best-qualified students subject to budget and capacity constraints. We show that student-proposing deferred acceptance is stable and strategy-proof for students, and that this mechanism implicitly allocates funding based on merit in colleges that report their preferences truthfully. Importantly, the set of stable allocations is large and the scope for manipulation by colleges is substantial, even in large markets. Successful manipulations consider applicants’ outside options (specifically, their financial need) when allocating funding. We corroborate our findings using data from Hungarian college admission where the centralized clearinghouse uses deferred acceptance. In particular, choosing a different stable allocation would increase the number of admitted students by at least 3%, and applicants from low socioeconomic backgrounds would benefit disproportionately.
1 Introduction

In recent years, a growing number of students are being assigned to schools through centralized clearinghouses. The success of such clearinghouses crucially relies on the use of a stable matching mechanism. Stability is also useful for predicting behavior in decentralized matching markets. Empirical and theoretical studies suggest that all applicants, save a handful, receive the same assignment in all stable allocations. This finding, that the set of stable allocations is small, has several implications. First, a designer who wishes to implement a stable outcome has limited scope for further design. Second, agents have little incentive to collect information on others. A closely related result is that incentives to misreport one’s preferences to the student-proposing deferred acceptance mechanism (henceforth DA) are minimal.

The above-mentioned results apply to settings where agents on both sides of a two-sided matching market (e.g., students and schools, men and women, etc.) have preferences over potential partners from the other side. However, the environments studied and designed by economists are often more com-

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1 See Roth and Xing (1994) and Roth (2002).
2 See Banerjee et al. (2013).
3 A large portion of the literature on the theory of two-sided matching markets is motivated by the potential multiplicity of stable allocations. Examples include studies of the structure of the set of stable allocations (Knuth, 1976), of fair stable allocations (Teo and Sethuraman, 1998), of the extent to which it is possible to improve the allocation of under-demanded hospitals or to increase the number of assigned doctors (Roth, 1986), and of incentives (Roth, 1982; Sonmez, 1999; Ehlers and Massó, 2007).
4 Unless otherwise specified, DA refers to the student-proposing version of deferred acceptance (Gale and Shapley, 1962).
5 Truthful reporting to the student-proposing DA mechanism is a weakly dominant strategy for students, and there is no stable matching mechanism that makes truthful reporting dominant for both sides of the market (Dubins and Freedman, 1981; Roth, 1982). Numerous studies have analyzed the optimal behavior of schools when the student-proposing DA mechanism is in place (examples include Sonmez, 1997; Roth and Rothblum, 1999; Ehlers, 2004; Konishi and Ünver, 2006; Coles, Gonczarowski and Shorrier, 2014; Azevedo and Budish, 2018) show that in large markets it is safe to report one’s true preferences to DA.
plex. For example, in college admissions markets, universities often offer admission to several study programs and multiple levels of financial aid. These more complex environments are studied in the matching-with-contracts literature (Hatfield and Milgrom 2005). Much of this literature focuses on identifying conditions under which DA remains stable and strategy-proof for students. The motivating question of this paper is: Do the findings on the size of the set of stable allocations and the good incentive properties persist in such environments?

They do not. We study a natural extension of the Gale and Shapley (1962) matching market model (without contracts), which captures the structure of preferences observed in centralized college admissions markets. We observe that under DA, financial aid decisions are based on merit (Theorem 1), and thus ignore students’ outside options. While several centralized college admissions clearinghouses use variants of DA and thus allocate financial aid based on merit, in all the instances we are aware of the mechanism was introduced prior to the introduction of multiple levels of aid, and we therefore do not know if the choice to allocate financial aid based on merit was intentional.

Based on this observation, we show that in large college admissions markets the expected fraction of students and colleges that have multiple stable allocations does not vanish as the size of the market grows large, and the same is true for the fraction of colleges that can successfully manipulate DA. Furthermore, the manipulation we identify takes a simple form that can be interpreted as colleges exercising their local market power over price-insensitive students by offering need-based – rather than merit-based – financial aid. Our proofs use a natural extension of the large (matching-without-contracts) market model of Kojima and Pathak (2009), but they have clear analogues in other models of large two-sided matching markets.

Empirically, we corroborate the predictions of the model using data from centralized college admissions markets, where variants of DA are in use. We show, based on reported preferences, that in Hungary, thousands of students have multiple stable allocations. Moreover, we show that switching from the DA outcome to another stable allocation would increase the number of stu-

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6The theory of matching with contracts has many other applications, such as the allocation of cadets to military branches (Sönmez 2013, Sönmez and Switzer 2013), school choice with dormitories or "special classes" (Wang and Zhou 2018), and entry-level labor markets (Niederle 2007, Dimakopoulos and Heller 2019). While the focus of this paper is on college admissions markets, our results also apply to these other environments.

7We are grateful to Joel Sobel for suggesting this interpretation of our result.
dents accepted to colleges in this country by more than 3% (approximately 2,000 students), and that colleges could substantially improve the quality of their incoming cohorts by using different admissions criteria. Rural applicants and applicants from lower socioeconomic backgrounds would benefit disproportionately.

Our findings suggest that answering “classic” questions in the theory of two-sided matching markets for the college admissions setting may be a fruitful direction. A natural question, for example, is: How does one find the stable allocation that matches the most students?

We begin with two examples illustrating the two key ideas behind our main results. The first example shows that the presence of multiple contractual terms may leave room for bargaining between a college and a student, even when the outcome must be stable. In the example, stability is not compromised if a college refuses financial aid to a student whose choice of college is not sensitive to the availability of financial aid, and who thus has no attractive outside options.

The second example shows that when colleges face a budget constraint, the outcome of the bargaining with one student affects the identity and the quantity of the other students the college can recruit. In the example, by refusing financial aid to price-insensitive students, the college relaxes its budget constraint, which allows it to recruit better, price-sensitive students. Our main theoretical result (Theorem 2) shows that there are many instances in which colleges can manipulate DA by reporting that price-insensitive students are ineligible for admission with financial aid, and shows that this manipulation results in a stable allocation that these colleges prefer. Intuitively, the reason is that under DA, financial aid decisions are based on merit, and thus ignore students’ outside options, and that the outcome of the bi-lateral bargaining with one student affects the college’s ability to bargain with other students. Readers who find this verbal description satisfactory may want to skip the examples and go directly to Section 1.2.

1.1 Examples

The following examples use the notation of the paper, but since the model has not yet been introduced, we provide a verbal description for each piece of notation.

Example 1. There are \( n \) colleges and \( m \) students. Colleges offer positions
with or without financial aid (formally, $T = \{0, 1\}$). Each college has a capacity of one regardless of funding ($q_0 = q_1 = 1$), and it prefers all student and financial aid combinations to the outside option of staying unmatched. For all colleges, accepting a student without financial aid is preferred to accepting the same student with financial aid. However, colleges care lexicographically more about the identity of the student than about funding (i.e., if accepting student $s$ without financial aid is preferred to accepting student $s'$ with financial aid, then accepting $s$ with financial aid is also preferred to accepting $s'$ with financial aid). The preferences of each college over students can be chosen arbitrarily. Students prefer admission with financial aid, but they too care lexicographically more about the identity of the college than about funding. Students also prefer any allocation to staying unmatched. Their preferences over colleges can also be chosen arbitrarily.

Claim 1. Under the above conditions, there are $\min\{m, n\}$ colleges and $\min\{m, n\}$ students with multiple stable allocations.

Proof. Since colleges have strict preferences and unit demand, there exists a stable allocation and the same agents are matched in all stable allocations (Hatfield and Milgrom, 2005). Since all agents find any allocation acceptable, stability implies that there cannot be unmatched agents on both sides of the market. Hence, $\min\{m, n\}$ colleges and students are matched in all stable allocations. Given a stable allocation, changing the terms while maintaining the identities of the contracting parties preserves stability.

Note that even if colleges know nothing more than the above description about the preferences of students, they have a simple manipulation for the student-proposing version of DA: by declaring all funded contracts unacceptable and reporting its true preferences over unfunded contracts, a college will be assigned the same student, but will not have to provide financial aid.\footnote{This holds under the assumption that students will use their weakly dominant strategy of reporting their preferences truthfully.} The driving force behind Example 1 is the near indifference of both sides of the market to funding, which implies that no agent has an attractive outside option. The requirement that all agents on the other side of the market be acceptable does not play an important role: in a model without this assumption, the only difference would be that $\min\{m, n\}$ is replaced with the cardinality of some stable allocation.

\footnote{\textsuperscript{8}This holds under the assumption that students will use their weakly dominant strategy of reporting their preferences truthfully.}
While the set of stable allocations in the market described in Example 1 is large in the sense that many agents (students and colleges) have multiple stable allocations, the proof relies on allocations in which the same agents contract with each other, and only the contractual terms differ. Given that we assumed that contractual terms are not particularly important for any of the agents (relative to the identity of their partner), the set of stable allocations may still be small in the sense that agents do not have “strong” preferences between stable allocations, and in the sense that the same agents are matched in all stable allocations. In fact, the latter must hold whenever colleges are interested in recruiting one student at most (Hatfield and Milgrom, 2005).

In the following example with one college, students differ in the importance they attribute to financial aid, and the college can accept more students than it can fund. The example highlights several differences between our college admissions environment and the one studied by Gale and Shapley (1962), where colleges offer students only one package of contractual terms. Notably, in our environment there is no student-optimal stable allocation, and the number of students attending college differs between stable allocations.

**Example 2.** There is one college \(C = \{h\}\) with two seats, but only one scholarship available \(T = \{0, 1\}, q^0_h = 2, q^1_h = 1\), and two students, \(S = \{r, p\}\). It may help to think of \(r\) (she) as a “rich” applicant and \(p\) (he) as a “poor” applicant. In subsequent examples \(h\) will be the “special” college whose perspective we take, and other colleges will be denoted by \(c\).

The rich applicant, \(r\), also happens to be a better “fit” with college \(h\) (using the notation of the model, \(r \gg_h p\)). The college prefers to accept the best “fit” students, and to fill its capacity. The college’s preferences over acceptable allocations are summarized as follows:

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\{(r, h, 0), (p, h, 0)\} \succ_h \{(r, h, 1), (p, h, 0)\} \succ_h \{(r, h, 0), (p, h, 1)\} \succ_h \{(r, h, 1)\} \succ_h \{(p, h, 0)\} \succ_h \{(p, h, 1)\} \succ_h \emptyset,
\]

where 1 (0) indicates admission with (without) financial aid. Applicant \(r\)’s preferences are \(r \succ_r (r, h, 1) \succ_r (r, h, 0) \succ_r \emptyset\). That is, she prefers to receive financial aid, but is willing to attend \(h\) even if she does not get it. The poor applicant, \(p\), is only interested in admission with financial aid. Thus, his preferences are summarized by \(p \succ_p (p, h, 1) \succ_p \emptyset\). Under these preferences, there are two stable allocations, \(\{(r, h, 1)\}\), which is the result of the student-proposing DA algorithm, and \(\{(r, h, 0), (p, h, 1)\}\).
Notably, the two allocations have different numbers of assigned students. Moreover, the outcome of DA is not the stable allocation most preferred by both students. In fact, a student-optimal stable allocation does not exist.\footnote{To see that there is no stable allocation that is most preferred by both students, note that the only allocation that both students weakly prefer to the above-mentioned stable allocations has both of them receiving financial aid, but this allocation is not acceptable to the college. \cite{hassidim2019} use this property to show that this matching-with-contracts market cannot be embedded in a matching-with-salaries market as in \cite{echenique2012}.}

### 1.2 Overview of the results

Our model of college admissions markets is a natural extension of the \cite{kojima2009} model of large two-sided matching markets (without contracts).\footnote{\cite{kojima2009} build upon the earlier model of \cite{immorlica2015} for large one-to-one matching markets (without contracts). We refer to our model as an extension of \cite{kojima2009} since some of our results require colleges to have capacities larger than one, and as we follow their notation and conditions closely.} In their model, while the number of schools is large, each student finds a small number of them acceptable. We augment their model by introducing different levels of financial aid from the same college. We assume that colleges face a constraint on the number of financial aid packages at each level, but otherwise have no strong preferences over the amount of financial aid they provide and over the identities of the funded students in a given cohort. We also assume that whenever an applicant finds a certain college acceptable under some terms, she prefers to attend that college under a more generous financial aid package.

The assumption on colleges’ preferences is consistent with the choice functions used by Hungarian colleges \cite{bior2012}, and reflects the reports of departments participating in the Israeli Psychology Master’s Match (IPMM; \cite{hassidim2017}). Although we show in Appendix D that this assumption could be substantially relaxed, we choose to focus in the body of the paper on the case that colleges have no strong preference for the distribution of aid to highlight that colleges have an incentive to be strategic in the allocation of financial aid even in the absence of direct incentives. Similarly, \cite{caniglia2020} choose to ignore social equity arguments in their analysis of Franklin & Marshall College’s move from merit- to need-based financial aid, saying that social equity arguments against merit aid complement their argument.
The assumptions on students' preferences reflect many features of the preferences reported to two centralized college-admissions matching-with-contracts markets: Hungarian college admissions and the IPMM. We show in Appendix C that the assumption that students always prefer one type of contract over another can be substantially relaxed as well (e.g., different contracts could correspond to different majors over which preferences are heterogeneous). We make this assumption to ensure that our results do not rely on configurations of preferences that we find unreasonable in many applications. Additionally, our results have clear analogues for other models of large matching markets where applicants are not assumed to be interested only in a limited number of schools (see Appendix G).

We prove that the expected fractions of applicants and of colleges whose assignment is different across stable allocations are large (non-vanishing in large markets), and that the same holds for the fraction of colleges that can successfully manipulate DA when all other agents are truthful. Furthermore, different stable allocations may result in substantially different numbers of students admitted to college.

We corroborate our theoretical predictions using administrative data from both the IPMM and the Hungarian market. We find that in Hungary, choosing a different stable allocation would increase the number of students admitted to college under the current student-proposing algorithm (about 60,000 a year) by more than 3%. Moreover, colleges could successfully manipulate DA and “poach” talent from their competitors. Our findings stand in sharp contrast to those of Roth and Peranson (1999), who find that only about 0.1% of approximately 20,000 applicants to the National Residency Match Program (NRMP) in the early 1990s would have received a different assignment had the algorithm been changed from hospital-proposing to applicant-proposing.

To provide intuition, we highlight the main differences between our model and that of Kojima and Pathak (2009), who find that under DA, when all agents are truthful, schools’ incentives to misrepresent their preferences are minimal, and the set of stable allocations is small. Kojima and Pathak attribute their results to the “vanishing market power” of schools when the student-proposing version of DA is used. Namely, even knowing others’ pref-

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11 The IPMM is strategy-proof for applicants. Like most real-life implementations of DA, the Hungarian college admissions mechanism is not, strictly speaking, strategy-proof (Pathak and Sonmez, 2013; Shorrer and Sóvágo, 2017). Outside of the matching literature, Avery and Hoxby (2004) make similar assumptions on students’ preferences. They too find empirically that students apply to a limited number of colleges.
ference reports, a school is highly unlikely to be able to strategically reject a student and as a result receive a proposal from another, preferred student, a necessary condition for the existence of a profitable manipulation in their setting.

By contrast, the driving force behind our results is the presence of market power that does not vanish. Given a stable allocation, colleges have local market power over students who receive financial aid, but who have no outside option that they prefer to a contract with the same college at a lower level of financial aid. An extreme case is when students rank all contracts with the same college consecutively. In this case, the college can “price discriminate” by offering the lowest acceptable level of financial aid to such students (as illustrated in Example 1), and the freed-up funds can then be used to recruit price-sensitive students (as illustrated in Example 2). Since students’ outside options depend on their preferences and on the behavior of other agents (students and colleges), information on others is crucial for colleges in order to successfully implement this strategy.

The presence of market power in our large-market model is the result of colleges offering several contracts that the same students tend to be interested in. In contrast to the environment without contracts, a college can successfully manipulate DA by generating new proposals only from students who have already proposed to the college. It is sufficient that a strategically rejected student apply for admission with a lower level of financial aid, as this would relax the college’s budget constraint, and allow it to accept a price-sensitive student it would have otherwise had to reject.

Our results shed light on the policy debate around market power in higher education (e.g., Hoxby, 2000). This literature gained traction after the Department of Justice brought an antitrust case against a group of elite colleges for sharing prospective students’ financial information and coordinating their financial aid policy. MIT contested the charges, claiming that this practice prevents bidding wars over the best students and thus frees up funds to support needy students, and that MIT does not profit financially from this practice (DePalma, 1992). In 1994, Congress passed the Improving America’s

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\[1\] In the context of large one-to-one matching markets without contracts, Immorlica and Mahdian (2015) have shown that certain regularities in preferences can lead to a large set of stable matchings. In this sense, the first part of our argument, namely, the existence of market power, can be interpreted as showing that multiple contracts in the same college generate such regularities. The second part of our argument relies crucially on colleges being interested in more than one student.
Schools Act, whose section 568 permits some coordination and the sharing of information between institutions with a need-blind admissions policy. Our model provides theoretical support to MIT’s arguments. We show that even in the absence of a motive to increase profit, colleges have an incentive to apply market power in order to improve the quality of their incoming cohorts, and that the consequences for students are heterogeneous. In particular, the direct effect is that some (needy) students gain and other (wealthy) students lose. Furthermore, the model illustrates how information about students’ alternatives can facilitate such behavior. Our findings are consistent with those of Caniglia and Porterfield (forthcoming) who analyze Franklin & Marshall College’s move from merit to need based financial aid.

Finally, we make a technical contribution. We show that, in contrast to the environment studied by Gale and Shapley (1962), in our environment a college may have multiple stable allocations but not be able to manipulate DA.

1.3 Related literature

Our study is most closely related to papers studying the size of the set of stable allocations in two-sided matching markets. There are many ways to think about the size of this set, and thus multiple notions of “smallness.” One such notion is where the same agents are matched across all stable allocations. This result is part of the rural hospital theorem proved by Roth (1984a, 1986) for the case of many-to-one markets with responsive preferences, and extended by Hatfield and Milgrom (2005) to environments with contracts where programs’ preferences meet certain conditions.

More recently, in 2013, a session in the annual winter meeting of the Council of Independent Colleges on how to end bidding wars for students and curtail the use of generous merit-aid packages led to a new DOJ investigation (which was subsequently closed). The National Association of Independent Colleges and Universities has recommended to Congress to consider providing a temporary antitrust exemption for private, nonprofit colleges, allowing them to share information and coordinate their financial aid policies. For more details see: https://www.naicu.edu/policy-advocacy/student-aid/anti-trust.

Caniglia and Porterfield (forthcoming) find that reallocating financial aid to needy students had no effect on the probability that non-needy students accept an offer of admission, but that it had a sizable effect on the probability that needy students accept such offer. This allowed the college to improve the “quality” of the incoming cohorts, and to diversify its student body.
An alternative notion of smallness has to do with the number of agents who receive a different assignment under different stable allocations. Early studies focus on marriage markets with the same number of men and women, where all members of the opposite sex are acceptable. In a random instance of such a market where strict preferences are drawn independently and uniformly at random, the set of stable allocations is typically large in this sense (Pittel, 1989, 1992).

Roth and Peranson (1999) show that the set of stable allocations in the NRMP is small in this sense. They attribute their finding to the market being large and students ranking only a small number of residency programs, and provide simulation evidence in support of their theory. This finding, which Roth and Peranson (1999) refer to as “core convergence,” is later proved theoretically by Immorlica and Mahdian (2015), Kojima and Pathak (2009), and Storms (2013), in increasingly general environments. Under an additional regularity condition, Kojima and Pathak (2009) prove that truthful reporting to DA is an approximate Bayesian Nash equilibrium. Ashlagi, Kanoria and Leshno (2017) show that the set of stable allocations is typically small even when all members of the opposite sex are acceptable, as long as the numbers of men and women is not exactly equal. Azevedo and Leshno (2016) study a model with a finite number of schools and a continuum of students, and find that generically there is a unique stable allocation.

Another notion of smallness is that the difference in utility between different stable allocations is small for all (or most) agents. Holzman and Samet (2014) find that the set of stable allocations in marriage markets is small in this sense when preferences are correlated. Lee (2016) allows for correlation in preferences through a common-value component and finds that under mild conditions on the distribution of preferences the set of stable allocations is small, and so are incentives to misreport under DA in marriage markets.\footnote{Coles and Shorrer (2014) show that even under incomplete information the exact best response of schools under DA can be substantially different from truthful reporting.}

In Section 3 we show that the set of stable allocations of large college admissions markets is large relative to all three of the above-mentioned senses of smallness. In Appendix A we show that the set of stable allocations in
college admissions markets coincides with the weak-domination core, which is contained in the (strict-domination) core. Thus, we show that the core in college admissions markets does not “converge.”

Our model of large college admissions markets builds upon the model of Kojima and Pathak (2009), which has been criticized by several authors for only guaranteeing slow rates of convergence (e.g., Kadam, 2014), and for generating some unrealistic predictions (e.g., Lee, 2016). These critiques do not apply in our case. To the contrary, it is well known that relaxing the assumptions of Kojima and Pathak (2009) may lead to a large set of stable matchings even in the absence of contracts. And our results have clear analogues in other models of large matching markets that are not subject to these critiques (Appendix G).

Our findings complement those of Azevedo (2014), who studies competition in quantities (i.e., capacity) in the Azevedo and Leshno (2016) environment. Azevedo (2014) finds that when schools are small compared to the rest of the market, they have no incentive to reduce their capacities. When schools are large, they have an incentive to exercise their market power by reducing their capacities. In the environment that we study, colleges’ preferences and strategy spaces are more complex, and colleges are able to exercise local market power over interested students, even when their size is negligible relative to the rest of the market. Furthermore, the manipulations we study may increase the number of students assigned to the manipulating college.

Our paper is related to the growing literature on matching with contracts (e.g., Kelso and Crawford, 1982; Roth, 1984b; Fleiner, 2003; Hatfield and Milgrom, 2005; Hatfield and Kojima, 2010; Hatfield and Kominers, 2015; Hatfield, Kominers and Westkamp, 2015), which has been applied to study other questions related to college admissions (Abizada, 2016; Afacan, 2017; Aygün and Bó, 2016; Nei and Pakzad-Hurson, 2016; Westkamp, 2013; Yenmez, 2018). Pakzad-Hurson (2014) is technically related. Using our terminology, he shows that, under certain conditions, when students are assured to have market power over colleges that offer multiple majors (i.e., when colleges care lexicographically more about the identity of the student than about her major) there exists a stable and student-efficient matching. Several studies assert that colleges can use early decision policy in order to exert their market power over rich applicants and increase aid to poor applicants (Avery, Fairbanks and Zeckhauser, 2009; Ehrenberg, 2009; Kim, 2010).

Closely related to our paper are studies of reserve design in the context of school choice (Dur et al., 2018; Dur, Pathak and Sönmez, 2016). A key
observation in this literature is that applicants are indifferent between different seats in the same school, which implies, using our terminology, that schools have market power over all assigned students. The reserve-design literature focuses on the effect of different ways the mechanism can break these preference ties to form strict student rankings of contracts, while keeping the priorities at each seat fixed. By contrast, we study an environment where students have strict preferences over all contracts, and we concentrate on changes to colleges’ preferences.

In the simple one-to-one setting, a school has an incentive to misreport its preferences to the student-proposing DA mechanism if and only if the school has multiple stable allocations (Demange, Gale and Sotomayor, 1986). In many-to-one markets, only one implication is correct. Namely, given a profile of preferences, a school may have a unique stable allocation and still have an incentive to misrepresent its preferences to a DA mechanism (e.g., Kojima and Pathak, 2009). We show that in college admissions markets neither statement implies the other. Namely, unlike a school, a college may also have multiple stable allocations but no incentive to misrepresent its preferences to a DA mechanism.

The paper is organized as follows. Section 2 presents our model and discusses the differences between our college admissions environment and the environment studied by Gale and Shapley (1962). Section 3 presents the main theoretical result, and a proof of a special case that illustrates key ideas in the complete proof (which is in Appendix B). Section 4 presents the empirical evidence. Section 5 discusses the implications of our findings and proposes directions for future research.

2 Model

We use the many-to-one matching-with-contracts model of Hatfield and Milgrom to describe college admissions environments. There is a finite set of colleges, $C$, a finite set of students, $S$, and a finite set of contractual terms, $T$. A contract is a tuple $(s, c, t) \in S \times C \times T$ that specifies a student, a college, and the contractual terms that govern their relationship. In this paper, $t \in T$ will typically describe the level of financial aid. The set of all possible contracts, $X$, is a subset of $S \times C \times T$.

We denote by $X_i$, the set of all possible contracts that involve agent $i \in S \cup C$. Each agent, $i$, has strict preferences over subsets of $X_i$, which we
denote by $\succ_i$. We often follow the convention of omitting sets that are ranked lower than the empty set from the description of $\succ_i$.

An allocation is a subset $Y \subseteq X$. Given an allocation $Y$, we sometimes refer to $Y_i := Y \cap X_i$ as agent $i$’s allocation. An allocation $Y$ is individually rational if for any agent $i$ the entire set $Y_i$ is the most-preferred subset of $Y_i$.

Finally, we assume that all students prefer the empty set to any subset with cardinality strictly greater than 1. Given that our interest is in individually rational allocations only, this encodes our assumption that the market is a many-to-one matching market. An allocation $Y$ is feasible if $|Y_s| \leq 1$ for each student $s$.

Financial aid

We identify the set of contractual terms with a finite subset of $\mathbb{N}$, $T = \{0, 1, \ldots, |T| - 1\}$. Unless otherwise specified, we think of $t \in T$ as a funding level, and assume that for each student, $s$, and college, $c$, $(s, c, t) \succ_s (s, c, t')$ if and only if $t > t'$. As we discuss later, this modeling choice is not crucial for any of our results, but is made in order to avoid configurations of preferences that are deemed unreasonable in the empirical application.

We make several assumptions on colleges’ preferences. First, each college, $c$, is associated with a sequence of numbers, $\{q^c_t\}_{t \in T}$, such that if $t < t'$ then $q^c_t \geq q^c_{t'}$. The number $q^c_t$ represents a constraint on the number of students who can be accepted with funding level $t$ or higher. Each college, $c$, prefers the empty allocation to all allocations that violate $c$’s quotas, that is, that assign to $c$ more than $q^c_t$ students with a funding level $t$ or higher.

Second, colleges have generalized responsive preferences. Each college, $c$, has a master list, a complete order over $S \cup \{\emptyset\}$, denoted by $\succ_c$. Given an allocation $Y$, a contract $(s, c, t) \in Y_c$, and a contract $(s', c, t) \notin Y_c$, $Y_c \succ_c (Y_c \setminus \{(s, c, t)\}) \cup \{(s', c, t)\}$ if and only if $s \succ_c s'$. Moreover, if $Y_c$ does not violate $c$’s quotas, $Y_c \succ_c Y_c \setminus (s, c, t)$ if and only if $s \succ_c \emptyset$. In words, the college will accept an additional contract as long as the student is ranked higher than the empty set on the college’s master list, and the contractual terms will not cause a quota violation.

Finally, we assume that, as long as quotas are not exceeded, the allocation of funding is less important than the identities of incoming students. Formally, if $Y_c \succ_c \emptyset$ and $Y'_c \succ_c \emptyset$, and there exists a bijection $\phi : Y_c \rightarrow Y'_c$ such that $x \in X_s \iff \phi(x) \in X_s$ for all $x \in Y_c$ and all $s \in S$ (i.e., $Y_c$ and $Y'_c$ differ only in contractual terms), but no such bijection exists be-
tween \( Y_c \) and \( Y''_c \) (roughly, \( Y_c \) and \( Y''_c \) differ in contracting parties), then
\[ Y''_c \succ_c Y_c \iff Y''_c \succ_c Y'_c. \]
This assumption, which we relax substantially in Appendix D, is realistic insofar as funding terms and quotas are often exogenously given.

The special case of our model where \(|T| = 1\) corresponds to two-sided many-to-one matching-without-contracts markets with responsive preferences, the environment studied by Kojima and Pathak (2009).

**Discussion of the assumptions on preferences**

In our empirical analysis, different contractual terms correspond to different levels of financial aid. For this reason, we choose to focus in the main text on the case that all students share the same ranking over contracts in the same college. The only reason we make this assumption is to reassure the reader that our theoretical results do not rely on configurations of preferences that can be deemed unreasonable in the empirical application. In Appendix C we show that our results continue to hold when preferences with respect to contractual terms are heterogeneous, which is the case, for example, when the same department offers multiple study tracks that are not naturally ranked, or when a limited number of dormitory units is available and students differ in their desire to live in a dorm.

Our assumption on capacities allows for what the NRMP calls “reversions,” i.e., donations of unfilled positions from one track to another (Roth and Peranson 1999; Niederle 2007). Importantly, it allows a college to fill its capacity with unfunded students and freely dispose of scholarships, as is the case in Hungary and in the IPMM, as well as in high-school choice in China. This assumption may be less appropriate when contracts correspond to different tracks in the same program. However, all of our results hold in an alternative model in which each college has a constant capacity for each of the contractual terms.

According to our assumptions, colleges do not have a financial aid budget that they are free to spend in any way the choose. Instead, we assume that there is a finite menu of alternatives (funding levels), and allow contracts with higher levels of funding to be “reverted” to contracts of lower levels. This modeling choice is realistic in settings where funding terms and quotas

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17 The (manipulable) mechanism used for high-school choice in China offers schools the ability to give admission priority in exchange for a higher tuition, for a fraction of the seats that each school offers (Wang and Zhou 2018).
are exogenously given. For example, funding may come from the government in the form of a tuition waiver, as in Australia, Ukraine, and Hungary (Artemov, Che and He 2017; Kiselgof, 2011; Shorrer and Sovagó 2017). Funding level may also be set by law, as in Turkey (Akar, 2010), or the “college” in our model may represent a department that is free to make admissions decisions, but whose funding policy is set by the institution (see, e.g., Hassidim, Romm and Shorrer 2017). Centralization and the need for a simple bidding language may also lead for this structure of preferences. For example, in the American market for Genetic Counseling graduate programs, where many of the programs offer multiple funding levels and multiple locations, reported preferences take this form. Finally, different contracts may correspond to indivisible goods, such as housing, location, or academic track.

The assumption that, subject to quotas, colleges preferences depend more on the quality of the incoming cohort than they do on the distribution of financial aid is not unusual (e.g., Kim, 2010; Heo, 2017). As discussed in the previous paragraph, this assumption too is realistic where funding terms and quotas are set exogenously for the admission committee. In Appendix D we relax this assumption substantially. We choose to focus in the main text on the case that colleges do not have strong preferences with respect to the distribution of financial aid in order to highlight the fact that colleges have an incentive to apply local market power even in the absence of a motive to increase profit or to redistribute aid. Still, we require that colleges preferences are strict, to reassure the reader that our results are not driven by preferences (cf. Erdil and Ergin, 2008).

**Choice functions and stability**

Agent $i$’s preferences induce a choice function, $C_h: 2^X \to 2^{X_i}$, that identifies the subset of $Y_i$ most preferred by $i$ for any subset $Y$ of $X$. Formally, $C_h(Y) := \max_{Z \subseteq Y_i} \{Z \mid Z \cap Y_i\}$. An allocation $Y$ is unblocked if there does not exist a college, $c$, and a non-empty $Z \subset X_c \setminus Y$ such that $Z_i \subset C_h(Z_i \cup Y_i)$ for all $i \in S \cup C$. An allocation $Y$ is stable if it is individually rational and unblocked. In Appendix A we show that an allocation is stable if and only if it belongs to the weak-domination core, which is a subset of the (strict-domination) core (Roth and Sotomayor, 1990).

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18 The fact that preferences are strict assures that $C_h(\cdot)$ is a function. It also implies that $C_h(\cdot)$ satisfies the irrelevance-of-rejected-contracts condition (Aygün and Sonmez, 2013).
A remark on our choice to make assumptions directly on preferences, rather than on choice functions, is in order. It is well known that, in general, assumptions on choice functions can be less restrictive than assumptions on preferences. We choose to make assumptions directly on preferences since we think that this makes them more transparent. These assumptions also reflect our understanding of colleges’ preferences, based on our practical experience (e.g., Hassidim, Romm and Shorrer 2017).

2.1 Properties of college admissions environments

We begin this section by noting that we have not restricted colleges’ choice functions to feasible allocations (i.e., at most one contract per student). With this observation, it is easy to verify that the choice functions satisfy the hidden-substitutes condition of Hatfield and Kominers (2015), and that they meet the other conditions of their Theorems 1–3 that assure that DA yields a stable allocation and is strategy-proof for students.

Observation 1. The set of stable allocations is non-empty in college admissions environments. Furthermore, DA terminates in a stable allocation, and the mechanism it induces is strategy-proof for students.

Observation 1 lists some similarities between our model and the matching-without-contracts environment studied by Gale and Shapley. Critically for our main result, it guarantees the existence of a stable allocation and assures that DA terminates in one such allocation.

The following theorem formalizes our claim that under DA financial aid is distributed based on merit.

Theorem 1. Let \( S, C, T, \{\succ_c, \gg_c, q_t\}_{t \in T}, \{\succ_s\}_{s \in S} \) be a college admissions market, and let \( Y \) be the stable allocation that corresponds to the outcome of DA. If two students are assigned to the same college under \( Y \), then the one the college ranks higher receives (weakly) more financial aid. Formally, for all \( s \) and \( s' \) in \( S \), for all \( c \) in \( C \), and for all \( t \) and \( t' \) in \( T \), if \((s, c, t)\) and \((s', c, t')\) belong to \( Y \), then \( s \gg_c s' \) implies \( t \geq t' \).

Proof. Assume to the contrary that \( s \gg_c s' \) but \( t < t' \). Since students prefer higher levels of financial aid, it must be the case that \( s \) requested admission to \( c \) under \( t' \), and his offer was rejected before the algorithm terminated. At the point where this rejection occurred, \( c \) tentatively held contracts with financial...
aid levels $t'$ and higher from $q^c_t$ students who were all ranked higher than $s$. But since $s \succ^c s'$, this implies that a request of $s'$ for admission under $t'$ must be rejected before the termination of the algorithm, a contradiction. \[\]

We next highlight key differences between our model and the matching-without-contracts environment studied by [Gale and Shapley (1962)].

**Proposition 1.** A college admissions market may have no stable allocation that is most preferred by all students. Furthermore, different stable allocations may have different numbers of assigned students.

*Proof.* Follows from Example 2 in Section 1.1. \[\]

**Proposition 2.** i) In a college admissions market, a college may have multiple stable allocations and yet not be able to manipulate DA when all other agents are truthful. ii) Furthermore, it may have a unique stable allocation and yet be able to manipulate DA when all other agents are truthful.

*Proof.* The second part holds even in the absence of contracts (Kojima and Pathak, 2009). The first part follows from Example 3. \[\]

**Example 3.** We add to the (special) college $h$ from Example 2 another (community) college, $c$, with one seat and no scholarships available ($C = \{h, c\}, q^c_0 = 1, q^c_1 = 0$). The poor student’s first choice is the funded seat at $h$, and although he now prefers the unfunded seat at $h$ to staying unmatched, he finds the (cheaper) community college more attractive. The rich student prefers college $h$ to the community college under any funding terms. Formally, the students’ preferences are now

$$(r, h, 1) \succ_r (r, h, 0) \succ_r (r, c, 1) \succ_r (r, c, 0) \succ_r \emptyset,$$

and

$$(p, h, 1) \succ_p (p, c, 1) \succ_p (p, c, 0) \succ_p (p, h, 0) \succ_p \emptyset,$$

c’s preferences are

$$(r, c, 0) \succ_c (p, c, 0) \succ_c \emptyset,$$

and $h$’s preferences remain

$$\{(r, h, 0), (p, h, 0)\} \succ_h \{(r, h, 1), (p, h, 0)\} \succ_h \{(r, h, 0), (p, h, 1)\} \succ_h \{(r, h, 0), (p, h, 1)\} \succ_h \{(p, h, 0), (p, h, 1)\} \succ_h \emptyset.$$
There are two stable allocations: \{ (r, h, 1), (p, c, 0) \}, which is the result of the student-proposing DA, and \{ (r, h, 0), (p, h, 1) \}. The number of students attending each college is different between the two stable allocations. Again, the outcome of DA is not the stable allocation most preferred by all students. And the only allocation that both students weakly prefer to both the above allocations requires both students to be funded by the special college, which violates the college’s individual rationality constraint. It is easy to verify that, when all other agents are truthful, \( c \) cannot do better than the outcome of DA, as \( r \) must be assigned to \( h \) regardless of \( c \)’s strategy.

The fact that \( c \) cannot manipulate DA even though it has multiple stable allocations is not due to one of them being the empty allocation. We could have augmented the example by adding a third student, \( d \), that is only interested in the community college, and whom all colleges least prefer. The stable outcomes would be \{ (r, h, 1), (p, c, 0) \}, which is the result of the student-proposing DA, and \{ (r, h, 0), (p, h, 1), (d, c, 0) \}, but \( c \) would still have no incentive to misrepresent its preferences to DA when others are truthful.

The example also illustrates the role of outside options. The special college, \( h \), has market power over the rich student in the allocation that results from DA because she has no outside option that she prefers to the unfunded contract with \( h \). The special college does not have market power over the poor student in the second stable allocation since, if he is not offered funding, he prefers to attend another college that will accept him. Example 4 in Appendix E shows that given a stable allocation, a college may have market power over an assigned student who ranks other contracts between that student’s allocation and another contract with the college, as long as the colleges who are parties to these contracts are not interested (and thus the student cannot form a blocking coalition with them). This observation proves useful for our empirical analysis.

It is worth pointing out that in the previous example, if the student-proposing version of DA is used, the private college has a simple profitable manipulation of declaring that the rich student is not eligible for financial aid. This manipulation is also feasible in a large market where the wealth level of applicants is known, and nothing else is known about students’ preferences other than that financial aid does not alter rich applicants’ preferences between colleges.
3 Theoretical Evidence

By now it is clear that college admissions environments have some properties that distinguish them from markets without contracts. We now turn to address our main questions: How likely can a college manipulate the student-proposing DA mechanism, and how likely does a college (student) have multiple stable allocations? To this end, we introduce a random environment that generalizes the Kojima and Pathak (2009) model of large matching markets to college admissions environments.

3.1 Regular sequences of college admissions markets

Let a uniform random market be a tuple \( \tilde{\Gamma} = \left< S, C, T, \{\succ_c, \succ_c, q^c_t\}_{t \in T}, k \right> \), where \( k \) is an integer greater than one, and \( \succ_c \) represents college \( c \)'s strict preferences (which must be consistent with its master list, \( \succ_c \), and list of quotas, \( \{q^c_t\}_{t \in T} \)). A uniform random market induces a college admissions market by drawing students’ preferences randomly in the following way:

- Step 1: for each student independently, draw \( k \) different colleges from \( C \) uniformly at random.
- Step 2: for each student independently, draw uniformly at random an acceptable permutation over the \( k \times |T| \) possible contracts with the colleges that were drawn in Step 1, where an acceptable permutation satisfies our assumption that students prefer higher levels of funding. Set the realized permutation as the student’s preferences. Other contracts are not acceptable to the student.

For each realization of students’ preferences, a (non-random) college admissions market is obtained. Setting \( |T| = 1 \) yields the distribution of preferences in uniform Kojima–Pathak markets. For notational convenience, we maintain the assumption of uniform random markets until the end of this section. In Appendix [C] we show that our results continue to hold in a much broader class of preference distributions (Proposition [4]). The natural generalization of the preference structure studied in [Kojima and Pathak (2009)] falls within this class, as do cases where contracts are not naturally ranked (e.g., multiple study tracks), or where some fraction of the population considers certain levels of financial aid prohibitively low. In Appendix [D] we
show that our results continue to hold when colleges care about the identity of the recipients of financial aid, as long as this consideration does not always supersede the quality of these recipients.

A sequence of uniform random markets, denoted by $\{\tilde{\Gamma}^n\}_{n=1}^{\infty}$ where $\tilde{\Gamma}^n := \left< S^n, C^n, T^n, \{\succ_c, \gg_c, q^c_t\}_{t \in T^n}, k^n \right>$, is regular if there exist integers $k, l, \bar{q}$, and $\lambda$, all greater than one, such that:

1. $|C^n| = n$ for all $n$,
2. $k^n = k$ and $T^n = \{0, 1, ..., l - 1\}$ for all $n$,
3. $q^c_0 \leq \bar{q}$ for all $c \in C^n$ and all $n$,
4. for all $n$, $c \in C^n$, and $s \in S^n$, $s \gg_c \emptyset$,
5. for all $n$ and $c \in C^n$, there exist $t, t' \in T^n$ such that $q^c_t > q^c_{t'} > 0$, and
6. $\frac{1}{\lambda} n \leq |S^n| \leq \lambda n$, for all $n$.

Condition 1 assures that the number of colleges grows as the sequence progresses. Condition 2 assures that the number of contracts that students consider acceptable is uniformly bounded on the sequence. Condition 3 assures that the number of positions in each college is uniformly bounded across colleges and markets. Condition 4 assures that colleges find any student acceptable. These conditions are identical to those of Kojima and Pathak (2009).

Condition 5 is the key addition we make to their model. This condition assures, roughly, that each college faces a financial aid constraint on top of the capacity constraint in Condition 3. Finally, Condition 6 assures that the number of students does not grow much faster or much slower than the number of colleges. Kojima and Pathak (2009) require only the first half of this condition. We require the second half as well since we are interested in instances where a substantial fraction of colleges have multiple stable allocations, but in markets with a small number of students (who each find contracts with at most $k$ colleges acceptable) most colleges will not even be a party to any individually rational allocation. Omitting both the lower bound on $|S^n|$ and Condition 5 and requiring $|T| = 1$ yields the definition of a sequence of uniform Kojima–Pathak markets.
3.2 Main theoretical results

Theorem 2. Given a regular sequence of uniform random markets, there exists $\Delta > 0$ such that for each random market in the sequence

1. the expected fraction of students with multiple stable allocations,

2. the expected fraction of colleges with multiple stable allocations that can successfully manipulate DA when all other agents are truthful,

3. the expected fraction of colleges with multiple stable allocations that cannot successfully manipulate DA when all other agents are truthful,

4. the expected fraction of colleges with different numbers of assigned students in different stable allocations, and

5. the expected fraction of students who are matched in some stable allocation, but are unmatched in another

are all greater than $\Delta$.

Proof. To keep the focus on the crux of the argument we defer the detailed proof to Appendix B and in what follows we analyze the special case of $T^n \equiv \{0, 1\}$ (e.g., students are either fully funded or not funded), $q^c_0 = 2$ and $q^c_1 = 1$ for all $c \in C^n$ for all $n$ (i.e., each college has two seats and can offer one scholarship), and $|S^n| = 2|C^n|$ (i.e., there are as many students as there are college seats). We also defer the treatment of Part 5 of the theorem, which requires a more subtle argument.

Consider two students, $r, p \in S^n$, a college $h \in C^n$, and some other college $c \in C^n$. Let the event $E^n(r, p, h, c)$ denote the case where:

1. College $h$ ranks $r$ higher than $p$ on its master list. Formally, $r \succ_h p$.

2. The only students who find contracts with $h$ acceptable are $r$ and $p$. Formally, for all $s \in S^n \setminus \{r, p\}$ and all $t \in T^n$, $\emptyset \succ_s (s, h, t)$.

3. The only student who finds contracts with $c$ acceptable is $p$. Formally, for all $s \in S^n \setminus \{p\}$ and all $t \in T^n$, $\emptyset >_s (s, c, t)$.

4. The two contracts $r$ finds most desirable are $(r, h, 1)$ and $(r, h, 0)$, and $r$ prefers the first to the second. Formally, for all $z \in X_r \setminus X_h$, $(r, h, 1) >_r (r, h, 0) >_r z$. 

22
5. The two contracts $p$ finds most desirable are $(p, h, 1)$ and $(p, c, 1)$, and $p$ prefers the first to the second. Formally, for all $z \in X_p$ \{(p, h, 1), (p, c, 1)\}, $(p, h, 1) \succ_p (p, c, 1) \succ_p z$.

Note that in the event $E^n(r, p, h, c)$, the students $r$ and $p$ and the colleges $h$ and $c$ have two stable allocations: \{(r, h, 1), (p, c, 1)\} (which is their allocation in the outcome of DA with respect to the true preference profile), and \{(r, h, 0), (p, h, 1)\}. Also, note that college $h$ can successfully manipulate DA by declaring that allocations under which $r$ receives financial aid are not acceptable, but clearly college $c$ cannot do likewise. Furthermore, the two colleges have different numbers of students assigned to them in different stable allocations.

Given a college $h \in C^n$, there are $2n \cdot (2n - 1) \cdot (n - 1)$ possible selections such that $r \neq p$ and $c \neq h$. Half of these events have zero probability (when $p \gg r$). The probability of each of the other events is greater than

$$\left(1 - \frac{k}{n - 1}\right)^{4n} \times \frac{1}{k^n} \times \frac{1}{n} \cdot \left(1 - \frac{1}{k}\right) \cdot \frac{1}{n} = \frac{k - 1}{k^2 n^3} \cdot \left(1 - \frac{k}{n - 1}\right)^{4n},$$

where each term in the leftmost expression corresponds to an (independent) requirement from the definition of $E^n(r, p, h, c)$.

Let $E^n_h = \bigcup_{(r, p, c) \in S^n \times S^n \times C^n} E^n(r, p, h, c)$. We now bound $\Pr[E^n_h]$ from below. We first note that for different selections of $(r, p, c)$, the events $E^n(r, p, h, c)$ are disjoint. Therefore, using the bound derived above, the probability of the event $E^n_h$ is greater than

$$n \cdot (2n - 1) \cdot (n - 1) \cdot \frac{k - 1}{k^2 n^3} \cdot \left(1 - \frac{k}{n - 1}\right)^{4n},$$

and

$$\liminf_{n \to \infty} \Pr[E^n_h] \geq \frac{2(k - 1)}{k^2} \cdot e^{-4k}.$$

Parts 2 and 4 follow immediately. Part 3 follows from a similar argument, where $c$ is held fixed and the union is taken over selections of $r$, $p$, and $h$. Part 1 follows from Part 4 and from the fact that each student finds contracts with at most $k$ colleges acceptable.

\footnote{To be precise, the first term refers to both the second and third requirements, as they are not independent.}
3.3 Discussion of the theoretical evidence

Condition 5 in the definition of a regular sequence of uniform random markets entails two parts. First, it assures that colleges offer multiple levels of financial aid \((q^c_t \geq q^c_{t'} > 0)\). Second, it assures that the constraint on financial aid is different than the physical constraint on the number of seats \((q^c_t > q^c_{t'})\). This second part is what makes colleges choice functions violate the substitutes condition (Hatfield and Milgrom, 2005). In particular, it assures that colleges’ capacities are not 1. Economically, this part of the condition is what links the outcomes of the bilateral bargaining between the college and different individual students.

Parts 1 and 2 of the main theorem do not rely crucially on the second part of Condition 5. For example, if colleges prefer to provide lower levels of financial aid (maintaining the assumption that colleges care more about the identities of the incoming cohort), then any college whose assignment under DA when others are truthful is not empty can manipulate DA and get a preferred stable allocation that differs only in contractual terms. The proof is completely analogous to the argument in Example 1, where colleges had a capacity of 1 (and thus their choice functions were substitutable).

By contrast, Parts 3–5 of the main theorem strongly rely on the second part of Condition 5. To see this, note that the events they refer to cannot occur when colleges choice functions are substitutable (e.g., by the rural hospital theorem). In this sense, the latter parts of the theorem are more interesting, as they go beyond saying that in bilateral bargaining there are multiple transfers consistent with the core.

Theorem 2 shows that the core does not “converge” by establishing lower bounds on several measures of its size. These lower bounds are not tight—they are derived using very special events, and there are many other (disjoint) events that would work just as well. Such other events often do not require that the student over whom local market power is applied ranks two contracts with the same college consecutively (see Appendix E). Appendix F provides simulation evidence that, in markets of realistic sizes, the actual values of these measures are substantially larger than the theoretical bounds we derive. Our results also have clear analogues in other models of large two-sided matching markets such as Azevedo and Leshno (2016) (see Appendix G).

\(^{20}\)To see this, note that the college \(h\) in our leading example rejects \((p, h, 1)\) from the menu \{\(p, h, 1\), \((r, h, 1)\}\) but does not reject this contract from the larger menu \{\(p, h, 1\), \((r, h, 1), (r, h, 0)\}\).
4 Empirical Evidence

In this section we study the Hungarian college admissions market. We provide evidence in support of our assumptions on the demand structure (i.e., students’ rank-order lists), corroborate the predictions of our model, and assess the potential welfare implications. Appendix H contains additional empirical evidence from the Israeli Psychology Master’s Match.

4.1 Background

College admissions in Hungary have been controlled centrally and organized through a centralized clearinghouse since 1985. Each year, about 100,000 students apply to bachelor’s programs and approximately 60,000 are assigned. As is standard in Europe, prospective students must choose in advance a particular study program: a specific major at a specific institution (e.g., B.A. in applied economics at Corvinus University).

Citizens of Hungary and of other member states of the European Economic Area are eligible to receive up to six years (12 semesters) of state-funded education. However, the government limits the number of state-funded seats in each field of study. While only eligible applicants may apply for admission with state funding, unfunded positions are also available and are open to all.

Over the years, the mechanism used by the clearinghouse has changed several times. Since 2008, a variant of student-proposing DA has been in use. Prior to that, a variant of the college-proposing algorithm was in place. Both mechanisms endow applicants with field-specific priority scores based on a weighted average of several variables (mainly matriculation exam scores and GPA in the 11th and 12th grades, but also some credit for disabled, disadvantaged, or gifted applicants). The weights in the formula differ for different fields of study.

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21 For more details see Biró (2012) and references therein.
22 In 2013, tuition ranged from $2,000 to $23,000 for three years, with a mean (over programs) of $4,500 and a median of $3,800. Many institutions grant funded students priority in access to subsidized housing and other amenities. The per capita GDP of Hungary in 2013 was $10,300.
23 Under the college-proposing version of DA, each slot is regarded as a separate program and the program-proposing variant of DA is used with different slots in the same college using the same ranking over students.
Since decisions about financial aid and admission are made simultaneously, and as the availability of funding may play a critical role in applicants’ decisions between programs, applicants are allowed to submit rank-order lists (ROLs) ranking any number of contracts (program and funding-level combinations). For example, an applicant may submit an ROL that ranks three contracts with two programs: 1) a funded B.A. in applied economics at Corvinus University, 2) a funded B.Sc. in agricultural engineering at the University of Debrecen, and 3) an unfunded B.A. in applied economics at Corvinus University. Applicants who wish to submit an ROL that ranks more than three programs (corresponding to up to six contracts) are required to pay a fee (about $7 per additional program on the ROL). This feature implies that truthful reporting is not a dominant strategy in the Hungarian mechanism [Haeringer and Klijn 2009]. But given a set of programs on a list, it is dominated not to rank all acceptable contracts with these programs truthfully. To be clear, applicants who apply for particular funding terms do not receive precedence; each field chooses the highest-priority students available to them subject to the capacity and funding constraints.

After the match results are realized, applicants are informed of their placement, and the priority-score cutoff for each contract is made public. The priority-score cutoff for a contract is equal to the score of the lowest-scoring student who was assigned to the particular contract. These statistics receive extensive media coverage in the days after the match results are published.

4.2 Data

Our analysis of the Hungarian college admissions process between 2009 and 2011 is based on Shorrer and Sóvágó (2017). Shorrer and Sóvágó merge data from four different sources. The main source of data is an administrative dataset containing much of the information available to the clearinghouse. This dataset includes each applicant’s ROL the priority score in each relevant contract, as well as the information that is required to recalculate it.

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24 We are abusing terminology here, since the formal definition of a contract includes the student’s identity.

25 We mostly rely on their Appendix F.

26 The Hungarian Higher Education Application Database (FEIWI) is owned by the Hungarian Education Bureau (Oktatasi Hivatal). The data were processed by the Hungarian Academy of Sciences Centre for Economic and Regional Studies (HAS-CERS).

27 The dataset reports the first 6 contracts on an applicant’s ROL as well as the applicant’s allocation, in case it was ranked lower. It also specifies the number of contracts on
(i.e., academic performance in relevant exams and whether the applicant has a certified disadvantaged status).

The administrative data on students who applied during their senior year of high school is merged with the National Assessment of Basic Competencies (NABC) dataset based on demographic variables. The NABC measures numeracy and literacy skills in a standardized way. Since 2008 it has covered all students in the 6th, 8th, and 10th grades who attended school on the day of the exam (prior to 2008 it covered a sample). The NABC dataset includes self-reported survey measures of socioeconomic status (e.g., parental education, home possessions, etc.). Shorrer and Sóvágó (2017) construct an NABC-based socioeconomic-status (SES) index, following Horn (2013). This index consists of three subindices: an index of parental education, an index of home possessions (number of bedrooms, cars, books, computers, etc.), and an index of parental labor market status.

Shorrer and Sóvágó’s dataset also includes microregion-level annual unemployment rates published by the National Employment Service in 2008, with a territorial breakdown consisting of 174 units. Finally, it includes the per capita gross annual income for all 3,164 localities for each year of the sample, calculated based on information published by the Hungarian Central Statistics Office.

4.3 Student rank-order lists

We now ask: Is the data consistent with the assumptions of our model on the distribution of student ROLs? Namely, are ROLs “short” (Roth and Peranson, 1999), and are applicants from lower socioeconomic backgrounds more likely to rank funded positions only, as our leading example suggests? The answer is positive.

As reported by Shorrer and Sóvágó (2017), 93.6% of the ROLs rank up to six contracts, and 99.1% of the ROLs are shorter than 10 contracts. Table 1 compares the characteristics of applicants who submitted ROLs ranking funded contracts exclusively with those of applicants who submitted ROLs ranking both funded and unfunded contracts. As one may expect, on average each ROL. In 93.6% of the cases, we observe the complete ROL.

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28 The objectives of the NABC are similar to those of the OECD Program for Interna-
tional Student Assessment (PISA). The NABC-based socioeconomic-status index resem-
bles the PISA economic, social, and cultural status (ESCS) indicator.

29 The third group, applicants who submitted ROLs ranking only unfunded contracts, is
Applicants with stronger academic performance were more likely to submit an ROL ranking funded contracts exclusively. This finding can be explained by these applicants expecting to be able to gain admission to a funded contract. In case they are not admitted with funding, they may prefer to re-take some exams and reapply the following year rather than pay tuition (Krishna, Lychagin and Robles, 2015), or they may be optimistic enough about their chances of admission to be (nearly) indifferent between their ROL and another ROL that ranks unfunded contracts (Chen and Pereyra Barreiro, 2015; Artemov, Che and He, 2017). Since SES is positively correlated with academic ability (Table 2), this pattern likely leads our analysis to understate the true scope for reallocating financial aid from wealthy applicants to needy ones.

4.4 Stable allocations and manipulations

We proceed to the main question of interest: Is the set of stable allocations in the Hungarian college admissions market large, and can colleges successfully manipulate the mechanism? To address this question, we need to take a stance on the units on colleges side that can coordinate their bargaining with students. The reason is that, currently, admission priorities for all contracts are set by the government, but we do not want to assess what the government can achieve by coordinating all contracts (which is arguably the set of individually rational allocations).

The natural candidates are programs, institutions (universities), or fields of study. The strategic unit we choose to focus on is a field of study, since this unit shares priorities and budget. The fact that different programs in the same field share a budget deems the other candidates inappropriate for this application. We further assume that the field’s priorities, which are small and includes ineligible students (e.g., applicants from countries outside the European Economic Area) and applicants who probably made mistakes (Hassidim et al., 2017; Rees-Jones, 2017; Rees-Jones and Skowronek, 2018).

30In the past, programs had more discretion in setting their admission priorities.

31Prior to 2008, each program had a separate quota of state-funded seats, and some discretion in determining the weights in the priority-score formula. Presumably, this meant that a higher number of students were facing a funding-quality tradeoff and, therefore, programs had greater scope for applying local market power.
Table 1: Characteristics of applicants who submitted ROLs with funded contracts only

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
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<tbody>
<tr>
<td>NABC-based SES index</td>
<td>-0.062***</td>
<td>-0.069***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.0017)</td>
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<td></td>
<td>(0.0017)</td>
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<tr>
<td>11th-grade GPA (1-5)</td>
<td>0.077***</td>
<td>0.094***</td>
<td>0.094***</td>
<td>0.093***</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(0.0023)</td>
<td></td>
<td>(0.0012)</td>
<td>(0.0012)</td>
<td>(0.0012)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Income (1,000 USD)</td>
<td></td>
<td>-0.038***</td>
<td>-0.039***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.0006)</td>
<td></td>
<td>(0.0006)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Unemployment (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.008***</td>
<td>0.008***</td>
<td></td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>(0.0002)</td>
<td>(0.0002)</td>
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<tr>
<td>Observations</td>
<td>78064</td>
<td>78064</td>
<td>284701</td>
<td>284701</td>
<td>284701</td>
<td>284701</td>
<td>284701</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.017</td>
<td>0.032</td>
<td>0.016</td>
<td>0.038</td>
<td>0.007</td>
<td>0.028</td>
<td>0.021</td>
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</tbody>
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* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: The regression coefficients are conditional on a year fixed effect and an indicator for missing values. Robust standard errors are in parentheses. The sample includes all ROLs, excluding those that ranked unfunded contracts only. In Columns 1 and 2 we restrict the sample to high-school-senior applicants, the population that was matched to the NABC data. The NABC-based SES index was matched to the main dataset based on 5 variables (year and month of birth, gender, school identifier, and four-digit postal code). It is normalized to have a mean of 0 and a standard deviation of 1 in the population of high-school students. Income stands for the per capita gross annual income in the locality where the applicant resided. Source: Shorrer and Sóvágó (2017)
Table 2: Academic ability and socioeconomic status

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Grade-11 GPA</th>
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<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>NABC-based SES index</td>
<td>0.085***</td>
</tr>
<tr>
<td></td>
<td>(0.0028)</td>
</tr>
<tr>
<td>Unemployment (%)</td>
<td>-0.005***</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
</tr>
<tr>
<td>Income (1,000 USD)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>78133</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.325</td>
</tr>
</tbody>
</table>

Notes: The regression coefficients are conditional on a year fixed effect and an indicator for missing values. Robust standard errors are in parentheses. The sample includes all ROLs, excluding those that ranked unfunded contracts only. In Column 1 we restrict the sample to high-school-senior applicants, the population that was matched to the NABC data. The NABC-based SES index was matched to the main dataset based on 5 variables (year and month of birth, gender, school identifier, and four-digit postal code). It is normalized to have a mean of 0 and a standard deviation of 1 in the population of high-school students. Income stands for the per capita gross annual income in the locality where the applicant resided. Source: Shorrer and Sóvágó (2017)
determined by the government, represent the true preferences of the field.

Fields of study can benefit by applying market power over students in multiple ways. One such way is to relocate students that are not sensitive to location in order to free up seats in highly demanded locations. A field of study can apply local market power in this way over any student who is only interested in programs in this specific field. We are not interested in coordination of this nature for several reasons. First, it requires making assumptions on the fields’ preferences for the location where students attend. Second, outside the scope of our model, one can imagine that this sort of coordination will lead to resistance from different units within the field.

Our interest lies in a different kind of coordination: refusing financial aid to students whose assignment will not change as a result of this refusal, and using the freed-up funds to recruit additional students. This boils down to assuming, for example, that the field of economics prefers that all the departments in the country will get a (weak) superset of their assigned students under DA, where the additional students are all desirable. Specifically, we even ignore the possibility of replacing some students with other, more desirable, students.

To summarize, we choose as a strategic unit the smallest unit that can rationalize the data (i.e., that is consistent with the fact that fields of study share a budget), and we make conservative assumptions on fields’ preferences. We assume only that each field is indifferent to the identities of the recipients of state funding, while keeping students’ placement fixed. Specifically, we do not take a stance on the field’s preferences with respect to transferring students from one program to another program within the same field.

Based on our conversations with admissions officers in multiple programs that participate in the IPMM, we feel very comfortable about the assumption that fields care about financial aid only insofar as it affects the composition of the incoming cohort. Furthermore, the assumption required for the empirical exercise is even weaker, namely, the assumption that, holding the assignment of students to programs in the field fixed, a field prefers to be assigned more students who meet the minimal requirements as long as quotas are not full, and thus to increase the total amount of tuition collected by each program in the field (from students and from the government).

As we have established in the previous sections, the set of stable allocations in a college admissions market does not, generally, admit a lattice structure with respect to same-side preferences. Thus, one cannot characterize it fully using standard methods. Moreover, data limitations complicate
the verification of stability (see Footnote 27). Instead, we take a different approach to assess by how much each field can improve its yield by applying market power over students who are placed in the field under DA when all colleges are truthful.

Given a field, $f$, and a year, $t$, our approach focuses on identifying students who receive financial aid in $f$ although the next-highest-ranked contract on their ROL whose priority score cutoff they pass is the unfunded contract with the same program (i.e., the same study track at the same institution). The set of such students, $MP^t_f$, is a set of students over whom $f$ has market power; i.e., $f$ can safely refuse their funding without creating new opportunities for them (or for others) to block.

There are 10,056 students who belong to some $MP^t_f$ in the sample period. They correspond to approximately 8% of the tuition waivers offered by the state in this period. Namely, when the financial aid offered in all other programs is held constant, about 8% of the waivers have no effect on the receiving students’ choice of program.\(^\text{32}\)

Next, we ask whether the field can improve the incoming cohort by refusing funding to students in $MP^t_f$. We are especially interested in knowing whether such behavior increases the total number of students attending some college.

To this end, we define the set $DB^t_f$ of students who stand to directly benefit if $f$ applies market power. Given a field, $f$, and a year, $t$, $DB^t_f$ is the set of up to $|MP^t_f|$ highest-priority-score year-$t$ applicants who were unassigned or assigned to a contract (not with $f$) that they ranked lower than the funded contract with a program in $f$ that had free capacity. We say that a program has free capacity if no student was rejected from the unfunded contract in this program.\(^\text{33}\)

We take a partial-equilibrium approach and assume that if $f$ has free capacity in some program, it can apply market power over members of $MP^t_f$ and use the freed-up funds to admit students in $DB^t_f$, so that the resulting allocation will be stable. A potential concern is that by “poaching” students

\(^{32}\)We are being conservative in that we do not even allow the field to rule out the possibility of receiving financial aid in another program in the same field.

\(^{33}\)Note that our approach makes the assumption that there is sufficiently free capacities in these programs. This assumption is mild for for two reasons. First, $DB^t_f$ typically does not contain many applications to the same program. Second, unfunded seats in programs with free capacity are offered in a secondary round, and capacities in this round are typically substantial.
from other programs, the field changes the outside options of students in MP^f in a way that takes away the field’s local market power over some of these students. It is unlikely that this issue has a significant effect on our findings. First, as we discuss in the next paragraph, the majority of students in DB^f are unassigned, so recruiting them does not effect the admission cutoffs in other programs. Second, there is no reason for students in MP^f to be marginal in any contract, and specifically in the contracts from which students in DB^f are “poached.” On the other hand, the partial equilibrium approach underestimates the number of students that will be affected by \( f \) applying market power, as it ignores indirect effects (on those who would take the place of “poached” students).

This analysis is performed separately for each field in each year. The number of students who stand to benefit directly is 9,463. (In some instances, mostly in STEM fields, the supply of funded seats is larger than the demand, and so freeing-up funds cannot help in recruiting additional students.) Of these, 5,886 students are not placed in any college in practice (i.e., under DA). Table 3 compares the characteristics of the groups. We find that the students who benefit from moving to another stable allocation (i.e., members of some DB^f), on average, come from lower SES relative to those who stand to lose from such a change (i.e., members of some MP^f), that they are more likely to live in a village, and that they are less likely to live in the capital, Budapest. They are also more likely to be female and to have graduated from a vocational high school. Mechanically, students who benefit from moving from the merit-based result of DA to another stable allocation have lower academic achievements than students who stand to lose from such a change.
Table 3: Characteristics of applicants in MP and DB

<table>
<thead>
<tr>
<th></th>
<th>DB</th>
<th>MP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>sd</td>
</tr>
<tr>
<td>Disadvantaged (dummy)</td>
<td>0.09</td>
<td>0.288</td>
</tr>
<tr>
<td>Unemployment rate (%)</td>
<td>7.95</td>
<td>4.668</td>
</tr>
<tr>
<td>Gross annual per capita income (1000 USD)</td>
<td>6.03</td>
<td>1.451</td>
</tr>
<tr>
<td>11th-grade GPA</td>
<td>3.77</td>
<td>0.777</td>
</tr>
<tr>
<td>Female</td>
<td>0.58</td>
<td>0.493</td>
</tr>
<tr>
<td>Secondary grammar school</td>
<td>0.64</td>
<td>0.479</td>
</tr>
<tr>
<td>Vocational school</td>
<td>0.32</td>
<td>0.468</td>
</tr>
<tr>
<td>Capital</td>
<td>0.14</td>
<td>0.348</td>
</tr>
<tr>
<td>County capital</td>
<td>0.21</td>
<td>0.405</td>
</tr>
<tr>
<td>Town</td>
<td>0.34</td>
<td>0.474</td>
</tr>
<tr>
<td>Village</td>
<td>0.31</td>
<td>0.463</td>
</tr>
<tr>
<td>Programs in ROL</td>
<td>3.31</td>
<td>1.295</td>
</tr>
<tr>
<td>Contracts in ROL</td>
<td>3.70</td>
<td>1.773</td>
</tr>
<tr>
<td>Observations</td>
<td>9,463</td>
<td></td>
</tr>
<tr>
<td>Unassigned under DA</td>
<td>5,886</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table compares the characteristics of applicants over whom some field can apply market power (members of some MP\(_j\)) with those of students who stand to benefit directly from moving to another stable allocation (members of some DB\(_j\)). The sample covers the years between 2009 and 2011. Each year, approximately 60,000 students are assigned to college through DA, of which approximately 70% receive funding. Source: Shorrer and Sövágó (2017)
5 Discussion

To conclude, we discuss one of the most well-known college admissions markets and highlight the relevance of our predictions to this market. College seats in Turkey are allocated centrally. Private colleges are obligated by law to offer a full scholarship in, at least, 15% of the seats they offer. Additionally, most institutions offer multiple contracts in the same program: a subsidized morning schedule, and a more expensive evening schedule (Akar, 2010). The centralized mechanism assigns students to college seats based on the college-proposing version of DA, as described in the seminal paper of Balinski and Sönmez (1999). A report by the World Bank states that the situation in Turkey “is akin to giving a large number of scholarships in each institution on the basis of merit” (Hatakenaka, 2006). The report also points that “an important group to target would be students from less privileged backgrounds, either in terms of income, regions, ethnicity or gender.” At the same time, private colleges are independently trying to improve the quality of their incoming cohorts by offering some scholarships outside of the centralized system. For example, Koc University’s Anatolian scholarship program focuses on students of low-socioeconomic backgrounds who rank marginally lower than the bar for a funded seat (thus, substantially higher than the admission bar for paying students).

Our theoretical findings predict these phenomena, and offer guidance on how to implement alternative policies. We have shown that using student-proposing DA in a college admissions market is akin to allocating financial aid based on merit (and this result readily extends to the college-proposing version of DA). We have established that the set of stable allocations in large college admissions markets is large, and that centralized two-sided college admissions markets that use DA leave much room for colleges to strategize. Specifically, colleges have an incentive to provide financial aid based on need rather than merit, even if they do not have preferences for equity or social justice and their only goal is to maximize the quality of their incoming cohort. In the Turkish context, this would mean not offering scholarships beyond the required quantity through the centralized mechanism, and instead targeting students who rank marginally lower than the bar for a funded seat and whose

choice of college is likely to change as a result of the availability of a scholarship. The incentive to target specific students, in turn, implies that colleges have an incentive to collect information on other agents in the market (both students and colleges).

These predictions stand in sharp contrast to those in markets without contracts. Since different stable allocations in college admissions markets do differ substantially, market designers are facing economically meaningful trade-offs, even when they are committed to stability. This suggests that studying the structure of the set of stable allocations in college admissions environments is a promising research direction. More importantly, new and improved mechanisms have the potential to influence and significantly promote policy goals with lasting economic impact, at both the individual and the societal level.

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A Stability and the Core

In this appendix we show that in college admissions environments, the set of stable allocations coincides with the (weak-domination) core (Roth and Postlewaite [1977]), which is a subset of the (strict-domination) core.\footnote{An allocation belongs to the (strict-domination) core if there does not exist a coalition of agents that can all get strictly higher utility by matching only among themselves. The}
Proposition 3. In college admissions environments, the set of stable allocations is equal to the weak-domination core.

Proof. The proof closely follows Proposition 5.36 in [Roth and Sotomayor (1990)]. Assume that the allocation $Y$ is not stable. Then there exist a college, $c$, and $Z \subset X_c \setminus Y$ such that $Z_i \subset Ch_i(Z_i \cup Y_i)$ for all $i \in S \cup C$. Thus, $Y$ is weakly dominated by the coalition consisting of $c$ and the collection of students involved in contracts in $Ch_c(Z_c \cup Y_c)$, through the allocation $Ch_c(Z_c \cup Y_c)$.

In the other direction, let $Y$ be an allocation not in the core. If it is not individually rational, we are done. Otherwise, it is weakly-dominated by another allocation $Y'$ via some coalition $A \subset S \cup C$. Hence, there is some college or student in $A$ that strictly prefers $Y'$ to $Y$. Since preferences are strict, there is at least one student $s \in A$ who gets a different allocation under $Y'$, and hence strictly prefers $Y'$ to $Y$ and to the outside option. Since $Y'$ is individually rational, $Y'_s$ consists of a single contract $(s, c, t)$ where $c \in A \cap C$. Since preferences are strict, $c$ strictly prefers $Y'$ to $Y$. Set $Z := Ch_c(Y \cup Y')$. Then $Z_c \subset Ch_c(Z_c \cup Y_c)$ by the irrelevance of rejected contracts [Aygü and Sönmez, 2013], since $Z = Ch_c(Y \cup Y')$ and $Z_c \cup Y_c \subset Y \cup Y'$. Additionally, if $i$ is a student involved in a contract in $Z$, then $Z_i \subset Ch_i(Z_i \cup Y_i)$ because $i \in A$ and thus $i$ prefers his allocation under $Y'$ to his (individually rational and hence singleton) allocation under $Y$. Thus, $Y$ is not stable. 

\[\Box\]

B Proof of Theorem 2

We now prove the theorem for the general case. This requires some additional notation. Given a college $c$ in $C^n$, let $\hat{t}_c$ be the maximal $t \in T^n$ such that $0 < q^t_c$. Similarly, let $\bar{t}_c$ be the maximal $t \in T^n$ such that $0 < q^t_c < q^{\bar{t}_c-1}_c$. The existence of $\hat{t}_c$ and $\bar{t}_c$ is assured by Condition 5 in the definition of a sequence of uniform random markets.

Given a college $h$ in $C^n$, an ordered selection of $q_{\hat{h}} + 1$ students in $S^n$, $(p, r_1, \ldots, r_{q_{\hat{h}}})$, and another college, $c$ in $C^n$, let $E^n(p, r_1, \ldots, r_{q_{\hat{h}}}, h, c)$ denote the event where:

1. College $h$ ranks lower-index $r_j$’s higher on its master list, and ranks all $r_j$’s higher than $p$. Formally, $r_j \succ_h r_i \succ_h p$ for all $1 \leq j < i \leq q^h$. 

Definition of the weak-domination core allows some members of the coalition to get the same utility.
2. The only students who find contracts with $h$ acceptable are the members of $\{p, r_1, \ldots, r_{q_h}^h\}$. Formally, for all $s \in S^n \setminus \{p, r_1, \ldots, r_{q_h}^h\}$ and all $t \in T^n$, $\emptyset >_s (s, h, t)$.

3. The only student who finds contracts with $c$ acceptable is $p$. Formally, for all $s \in S^n \setminus \{p\}$ and all $t \in T^n$, $\emptyset >_s (s, c, t)$.

4. All $r_j$’s prefer to be placed in $h$ under any terms to any contract with another college. Formally, for all $1 \leq j \leq q_h^h$, and for all $z \in X_{r_j} \setminus X_h$, $(r_j, h, 0) >_{r_j} z$.

5. The most desirable contracts for $p$ are $(p, h, |T| - 1), (p, h, |T| - 2), \ldots, (p, h, \tilde{t}_h)$, followed by $(p, c, |T| - 1), (p, c, |T| - 2), \ldots, (p, c, 0)$. Formally, for all $z \in X_p$, $z >_p (p, c, |T| - 1)$ if and only if $z \in \{(p, h, t) | t \geq \tilde{t}_h\}$, and $z >_p (p, c, 0)$ if and only if $z \in \{(p, h, t) | t \geq \tilde{t}_h\} \cup \{(p, c, t) | t > 0\}$.

Note that in the event $E^n_{(p, r_1, \ldots, r_{q_h}^h, h, c)}$, student $p$ and colleges $h$ and $c$ have multiple stable allocations. The stable allocation resulting from DA includes $(r_1, h, \tilde{t}_h)$ and $(p, c, \hat{t}_c)$. But another stable allocation involves a contract of the form $(p, h, t)$ for $t \geq \tilde{t}_h$ (and some $r_j$ receiving a lower level of financial aid at $h$). Note that college $h$ can successfully manipulate DA (e.g., by declaring that allocations under which $r_1$ receives financial aid are not acceptable), but clearly college $c$ cannot do likewise. Furthermore, the two colleges have different numbers of students assigned to them in different stable allocations.

The following lemmas are used to prove the different parts of Theorem 2. Lemma 1 is used for Parts 1–4. Lemmas 2 and 3 are used for Part 5.

**Lemma 1.** There exists $L > 0$ and $n'$, such that for all $n > n'$ and for each college $h \in C^n$, the event

$$E^n_{(p, r_1, \ldots, r_{q_h}^h, h, c)} = \bigcup_{(p, r_1, \ldots, r_{q_h}^h, c) \in S^n \times \cdots \times S^n \times C^n} E^n(p, r_1, \ldots, r_{q_h}^h, h, c)$$

has a probability bounded below by $L$.

**Proof.** For sufficiently large $n$, given a selection of $h$, there are at least $\binom{q_h^h}{q_{q_h}^h + 1}$ selections of $(p, r_1, \ldots, r_{q_h}^h)$ that meet the first condition. Given such a selection and a selection of one of the $n - 1$ other colleges $c \neq h$, the probability
that the other conditions are met is bounded below by

\[
\left(1 - \frac{k}{n - 1}\right)^{[2\lambda n]} \times \frac{1}{(n \cdot k^{|T|-1})^{q_{th}}^h} \times \frac{1}{n^2 \cdot k^{|T|}},
\]

where each term in this expression corresponds to an independent requirement from the definition of the event \(E_n(p, r_1, \ldots, r_{q_{th}}^h, h, c)\). Moreover, since, given \(h\), the events \(E_n(p, r_1, \ldots, r_{q_{th}}^h, h, c)\) are disjoint, the probability of their (disjoint) union is equal to the sum of their probabilities, and is therefore greater than

\[
(n - 1) \times \left(\frac{n}{\lambda}\right)^{[2\lambda n]} \times \left(1 - \frac{k}{n - 1}\right)^{[2\lambda n]} \times \frac{1}{(n \cdot k^{|T|-1})^{q_{th}}^h} \times \frac{1}{n^2 \cdot k^{|T|}},
\]

which converges to a positive constant (that depends on \(q_{th}^h\)) as \(n\) grows large. Since \(q_{th}^h < \bar{q}\) by Condition 3 in the definition of uniform random markets, \(q_{th}^h\) can take only finitely many values; hence, taking the minimum of the limits and subtracting some small \(\varepsilon\) suffices.

\[\square\]

Lemma 2. Given a profile of (complete) college master lists, at least \(\frac{1}{3}\) of the students are ranked between \(\frac{1}{4}|S|\) and \(\frac{3}{4}|S|\) in at least \(\frac{1}{4}\) of the lists.

Proof. To prove this combinatorial claim we use the probabilistic method. First, note that the fraction of students who appear in this half of the list of at least \(\frac{1}{4}\) of the lists is equal to the probability that this condition is satisfied by a student drawn uniformly at random. Let \(\chi_c(\cdot)\) denote the indicator variable that a student is in the top or bottom quarter of \(c\)’s list. Then, by Markov’s inequality,

\[
\Pr\left[\sum_{c \in C} \chi_c > \frac{3}{4} n\right] \leq \frac{n/2}{3n/4} = \frac{2}{3},
\]

where probabilistic statements are with respect to the uniform distribution over students. This completes the proof. \[\square\]

\[\text{To be precise, the second and third conditions are not independent. The first term of our bound applies to both of them simultaneously.}\]
Denote by \( M^n \subset S^n \) the collection of students who are ranked between \( \frac{1}{4}|S^n| \) and \( \frac{3}{4}|S^n| \) in at least \( \frac{n}{4} \) lists. Given a selection of colleges, \((h, c, c_1, \ldots, c_{k-1})\), and students, \( (p, r_1, \ldots, r_{q_0}^h, \{s_{c_i}^j\}_{j=1}^{\bar{q}}, \{s_{c_i}^j\}_{j=1}^{\bar{q}})^{k-1} \), let \( E^n(p, r_1, \ldots, r_{q_0}^h, h, c, c_1, \ldots, c_{k-1}, \{s_{c_i}^j\}_{j=1}^{\bar{q}}, \{s_{c_i}^j\}_{j=1}^{\bar{q}})^{k-1} \) denote the event where:

1. The student \( r_{q_0}^h \) belongs to \( M^n \).
2. Each college in \( \{h, c_1, \ldots, c_{k-1}\} \) ranks \( r_{q_0}^h \) between \( \frac{1}{4}|S^n| \) and \( \frac{3}{4}|S^n| \).
3. Each college \( c_i \in \{c_1, \ldots, c_{k-1}\} \) ranks all of the students \( \{s_{c_i}^j\}_{j=1}^{\bar{q}} \) among the highest \( \frac{1}{4}|S^n| \).
4. College \( h \) ranks lower-index \( r_j \)'s higher on its master list, and ranks \( p \) between \( r_{q_0}^h \) and \( r_{q_0}^h + 1 \). Formally, \( r_j \succ_h r_i \) for all \( 1 \leq j < i \leq q_0^h \) and \( r_{q_0}^h \succ_h p \succ_h r_{q_0}^h + 1 \).
5. The only students who find contracts with \( h \) acceptable are the members of \( \{p, r_1, \ldots, r_{q_0}^h\} \). Formally, for all \( s \in S^n \setminus \{p, r_1, \ldots, r_{q_0}^h\} \) and for all \( t \in T^n \), \( \emptyset \succ_s (s, h, t) \).
6. The only student who finds contracts with \( c \) acceptable is \( p \). Formally, for all \( s \in S^n \setminus \{p\} \) and for all \( t \in T^n \), \( \emptyset \succ_s (s, c, t) \).
7. All \( r_j \)'s prefer to be placed in \( h \) under any terms to any contract with another college. Formally, for all \( 1 \leq j \leq q_0^h \), and for all \( z \in X_{r_j} \setminus X_h \), \( (r_j, h, 0) \succ_{r_j} z \).
8. The most desirable contracts for \( p \) are \( (p, h, |T| - 1), (p, h, |T| - 2), \ldots, (p, h, t_h) \), followed by \( (p, c, |T| - 1), (p, c, |T| - 2), \ldots, (p, c, 0) \). Formally, for all \( z \in X_p \setminus \{(p, h, t) \mid t \geq t_h\} \), \( (p, c, |T| - 1) \succ_p z \), and \( z \succ_p (p, c, 0) \) if and only if \( z \in \{(p, h, t) \mid t \geq t_h\} \cup \{(p, c, t) \mid t > 0\} \).
9. The lowest-ranked \( r \)-student on \( h \)'s master list, \( r_{q_0}^h \), finds contracts only with \( (h, c_1, \ldots, c_{k-1}) \) acceptable, and prefers \( (r_{q_0}^h, c_i, |T| - 1) \) to \( (r_{q_0}^h, c_j, |T| - 1) \) if \( i < j \). Formally, for all \( u \in C^n \) and \( t \in T^n \),
u \notin \{h, c_1, \ldots, c_{k-1}\} \implies \emptyset \succ_{r_{q_0}^h} (r_{q_0}^h, u, t), \text{ and } (r, c_i, |T| - 1) \succ_{r_{q_0}^h} (r, c_j, |T| - 1) \text{ if and only if } i < j.

10. For each $c_i \in \{c_1, \ldots, c_{k-1}\}$ the only students who find contracts with $c_i$ acceptable are $r_{q_0}^h$ and the members of $\{s^j_{c_i}\}_{j=1}^{q}$. Formally, for all $i \in \{1, 2, \ldots, k-1\}$, for all $s \in S^n \setminus \{s^j_{c_i}\}_{j=1}^{q}$, $s \neq r_{q_0}^h$, and for all $t \in T^n$, $\emptyset \succ_s (s, c_i, t)$.

11. For each $c_i \in \{c_1, \ldots, c_{k-1}\}$ the members of $\{s^j_{c_i}\}_{j=1}^{q}$ prefer to be placed in $c_i$ under any terms to any contract with another college. Formally, for all $i \in \{1, 2, \ldots, k-1\}$, for all $1 \leq j \leq q$, and for all $z \in X_{s^i_{c_i}} \setminus X_{c_i}$, $(s^j_{c_i}, c, 0) \succ s^j_{c_i}, z$.

Note that in the event $E^n \left( p, r_1, \ldots, r_{q_0}^h, h, c, c_1, \ldots, c_{k-1}, \{s^j_{c_i}\}_{j=1}^{q} \right)$ the stable allocation that corresponds to the outcome of DA assigns students $\{r_1, \ldots, r_{q_0}^h\}$ to $h$, and student $p$ to $c$. But in another stable allocation, college $c$ and student $r_{q_0}^h$ receive no assignment, and students $\{p, r_1, \ldots, r_{q_0}^h-1\}$ are assigned to $h$, where one of the students in $\{r_1, \ldots, r_{q_0}^h\}$ receives a lower level of financial aid relative to the stable allocation that corresponds to the outcome of DA.

**Lemma 3.** There exists $L > 0$ and $n'$, such that for each $n > n'$, for each $s \in M^n$, the probability that the event

$$E^n_s := \bigcup_{r_{q_0}^h = s} E^n \left( p, r_1, \ldots, r_{q_0}^h, h, c, c_1, \ldots, c_{k-1}, \{s^j_{c_i}\}_{j=1}^{q} \right)$$

is greater than $L$.

**Proof.** The proof is analogous to the proof of Lemma 1. Let $r_{q_0}^h$ be a student in $M^n$. Conditional on the selection of

$$\left( p, r_1, \ldots, r_{q_0}^h, h, c, c_1, \ldots, c_{k-1}, \{s^j_{c_i}\}_{j=1}^{q} \right)$$
being valid, the probability of the event
\[ E^n \left( p, r_1, \ldots, r_{q^h_0-1}, h, c, c_1, \ldots, c_{k-1}, \left\{ \{s_{j_i}^i\}_{j=1}^{k-1} \right\}_{i=1}^{k-1} \right) \]

is bounded below by
\[
\left( 1 - \frac{k}{n - 2k} \right)^{2\lambda_n} \times \left[ \left( \frac{1}{k} \right)^{|T|q} \cdot \left( \frac{1}{n} \right)^{q^h_0} \right] \times \left[ \left( \frac{1}{k} \right)^{2|T|} \cdot \left( \frac{1}{n} \right)^2 \right] \times \left( \frac{1}{n} \right)^{k-1} \times \left[ \left( 1 - \frac{k}{n - 2k} \right)^{k\lambda_n} \cdot \left( \frac{1}{n} \right)^{q(k-1)} \right] \times \left( \frac{1}{k} \right)^{|T|}.
\]

This expression behaves asymptotically like \( \bar{C} \cdot \left( \frac{1}{n} \right)^{q^h_0 + \bar{q}(k-1) + k+1} \) for some positive constant \( \bar{C} \).

Next, we calculate a lower bound for the number of valid selections of \( \left( p, r_1, \ldots, r_{q^h_0-1}, h, c, c_1, \ldots, c_{k-1}, \left\{ \{s_{j_i}^i\}_{j=1}^{k-1} \right\}_{i=1}^{k-1} \right) \). Using Lemma 2, there are at least \( \left( \frac{|S^n|}{k} \right)^k \) different ways to choose \( (h, c_1, \ldots, c_{k-1}) \). Given this selection, there are at least
\[
\left( \frac{|S^n|}{k} \right)^{k-1} \left( \frac{1}{n} \right)^{q^h_0 + (k-1)\bar{q}} \left( n - k \right)
\]
valid ways to choose \( \left( p, r_1, \ldots, r_{q^h_0-1}, \left\{ \{s_{j_i}^i\}_{j=1}^{k-1} \right\}_{i=1}^{k-1} \right) \). Here, we use the fact that each college in \( \{h, c_1, \ldots, c_{k-1}\} \) ranks \( q^h_0 \) between \( \frac{1}{4}|S^n| \) and \( \frac{3}{4}|S^n| \), which implies the existence of \( \left( \frac{|S^n|}{4} \right)^k \) higher (lower) ranked students. Finally, there are \( n - k \) ways to choose \( c \).

To complete the proof, we note that for different selections of
\[
\left( r_1, \ldots, r_{q^h_0-1}, h, c, c_1, \ldots, c_{k-1}, \left\{ \{s_{j_i}^i\}_{j=1}^{k-1} \right\}_{i=1}^{k-1} \right)
\]
the events
\[ E^n \left( p, r_1, \ldots, r_{q^h_0-1}, h, c, c_1, \ldots, c_{k-1}, \left\{ \{s_{j_i}^i\}_{j=1}^{k-1} \right\}_{i=1}^{k-1} \right) \]
are disjoint. Thus, the probability of their union is equal to the sum of their probabilities and hence bounded below by
\[
\left[ \left( \frac{|S^n|}{k} \right)^k \cdot (n - k) \right] \times \left[ \bar{C} \cdot \left( \frac{1}{n} \right)^{q^h_0 + (k-1)\bar{q}} \cdot (n - k) \right].
\]

37 We use the term “valid” when we refer to conditions on colleges’ preferences, to emphasize that these are arbitrary (i.e., not assumed random).
which asymptotes to a positive constant, \( L > 0 \).

**Proof (of Theorem 2).** Parts 2 and 4 of the theorem follow directly from Lemma 1. Let \( \chi_{E^h_n} \) denote the indicator of the event \( E^h_n \). Part 3 of the theorem follows from the fact that the expectation of the random variable \( \sum_{h \in C^h_n} \chi_{E^h_n} \) increases linearly in \( n \), which implies that the expected number of colleges “playing the role of \( c \)” is large\(^{38}\). Part 1 of the theorem follows from Part 4 and from the fact that each student has at most \( k \) colleges she prefers to the outside option. Part 5 of the theorem follows from Lemmas 2 and 3, that together assure that at least a third of the students have probability greater than \( L > 0 \) to be assigned in one stable allocation but not in another. \( \Box \)

### C  WEAKER DISTRIBUTIONAL ASSUMPTIONS

In this appendix, we formalize the claim that Theorem 2 generalizes to a broad class of distributions over students’ preferences.

**Proposition 4.** Let \( \{\bar{\Gamma}^n\}_{n=1}^{\infty} \) be a sequence of uniform random markets, and let \( \{\tilde{\Gamma}^n\}_{n=1}^{\infty} \) be another sequence of random markets that differs only in the distribution of students’ preferences.\(^{39}\) Then, Theorem 2 holds for \( \{\bar{\Gamma}^n\}_{n=1}^{\infty} \) so long as there exists some \( \bar{C} > 0 \) and \( \alpha \in (0,1) \) such that for sufficiently large \( n \), at least a fraction \( \alpha \) of the events

\[
E^n \left( p, r_1, \ldots, r_{q_h^*}, h, c \right)
\]

and

\[
E^n \left( p, r_1, \ldots, r_{q_0^*}, h, c, c_1, \ldots, c_1, \ldots, c_{k-1}, \left\{ s_{c_i} \right\}_{j=1}^{q_j} \right)
\]

with valid selections (with respect to college preferences) are at least \( \bar{C} \) times as likely in \( \bar{\Gamma}^n \) as they are in \( \bar{\Gamma}^n \).

**Proof.** Follows immediately from our proof of Theorem 2. \( \Box \)

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\(^{38}\)This can be shown formally by changing the order of summation.

\(^{39}\)Below, we present a definition of a random market that makes restrictions on the distribution of students’ preferences. With a slight abuse of notation, we use the same term but do not make any restrictions.
Allowing the preference-generating process from the body of the paper to draw uniformly from all permutations (not just acceptable ones) satisfies the conditions of the proposition, as does allowing $k$, the number of colleges acceptable to each student, to be student-specific (but uniformly bounded). The conditions also hold in sufficiently thick regular sequences of random markets (defined below), a broad family of sequences that generalizes the structure studied in Kojima and Pathak (2009) to the college admissions setting.

Let a **random market** be a tuple $\tilde{\Gamma} = \langle S, C, T, \{\succ_c, \gg_c, q^c_t\}_{t \in T}, k, \kappa, \mathcal{D} \rangle$, where $k$ is an integer greater than one, $\kappa$ is a number in $(0, 1)$, $\succ_c$ represents college $c$'s strict preferences (which must be consistent with its master list, $\gg_c$, and its list of quotas, $\{q^c_t\}_{t \in T}$), and $\mathcal{D} := \{p_c\}_{c \in C}$ is a distribution on $C$. A random market induces a college admissions market by drawing, for each student independently at random, preferences in the following way:

- **Step 0**: Set $t = 1$, $A = \emptyset$, $B = \emptyset$, and let $R$ be an empty ROL.
- **Step $t = 1, \ldots, k|T|$:** If there are contracts with $k$ different colleges on $R$, proceed to step $t.2$. Otherwise, with probability $\kappa$ proceed to step $t.1$, and with probability $(1 - \kappa)$ proceed to step $t.2$.
  - **Step $t.1$:** Draw a college, $c$, according to $\mathcal{D}$. If $c$ is in $A \cup B$, repeat. Otherwise, append the most generous contract with $c$ to the ROL, add $c$ to $B$, and continue to step $t + 1$.
  - **Step $t.2$:** If $B$ is empty, continue to step $t.1$. Draw uniformly at random a college from $B$, $c \in B$. Append to the ROL $R$ the most generous contract with $c$ that does not appear on $R$. If the terms of this contract are the lowest in $T$, remove $c$ from $B$ and add it to $A$. Continue to step $t + 1$.
- **Step $k|T| + 1$:** Set $R$ as the student’s preferences, where other contracts are not acceptable.

A sequence of random markets, denoted by $\{\tilde{\Gamma}^n\}_{n=1}^\infty$, is **regular** if there exist integers $k, l, \bar{q}$, and $\lambda$, all greater than one, and $\kappa \in (0, 1)$ such that:

1. $|C^n| = n$ for all $n$,\n
---

\textsuperscript{40}Each randomization in the algorithm is independent.
2. \( k^n = k, \kappa^n = \kappa, \) and \( T^n = \{0, 1, ..., l - 1\} \) for all \( n \),

3. \( q^c_0 \leq \bar{q} \) for all \( c \in C^n \) and all \( n \),

4. for all \( n, c \in C^n, \) and \( s \in S^n, s \gg_c \emptyset \),

5. for all \( n \) and \( c \in C^n \), there exist \( t, t' \in T^n \) such that \( q^c_t > q^c_{t'} > 0 \), and

6. \( \frac{1}{n} \leq |S^n| \leq \lambda n \), for all \( n \).

A regular sequence of random markets, \( \{\tilde{\Gamma}^n\}_{n=1}^\infty \), is sufficiently thick if there exist \( \rho > 0, \omega \in (0, 1) \), and an integer \( n' \) such that for all \( n > n' \),

\[
\max_{c \in C^n} p^c_{\text{max}} \leq \rho, \\
\max_{\omega n} p^c_{\text{max}} < \rho,
\]

where \( \max_i \) is the \( i \)-th highest element in a set. This condition means that the most popular college is at most \( \rho \) times as popular as the \( (100 \omega) \) percentile college (i.e., the ratio of popularities does not grow without bound).

D Weaker Assumptions on Colleges’ Preferences

In this appendix, we relax the assumption that colleges care lexicographically more about the composition of the incoming cohort than they do about how financial aid is allocated among these students.

**Definition 1.** For each college, \( c \in C \), and student \( s \in S \), denote the percentile rank of student \( s \) on \( c \)'s master list by \( P^c_s := \frac{|\{s' \mid s' \gg_c s\}|}{|S|} \).

The choice functions described in the body of the paper can be represented as the argmax of the following utility function:

\[
u_c(Z) = \begin{cases} -\infty & \text{if } Z_c \text{ violates one of } c\text{'s quotas} \\ \sum_{(s,c,t) \in Z_c} (1 - P^c_s + \epsilon(s,c,t)) & \text{otherwise} \end{cases}
\]
where $\epsilon(s, c, t)$ are sufficiently small and assure that there are no preference ties. That we do not require feasibility makes transparent the fact that the choice function satisfies the hidden-substitutes condition of Hatfield and Kominers (2015), as well as the other conditions of their Theorems 1–3: the law of aggregate demand and the irrelevance-of-rejected-contracts condition.

Next, we consider choice functions induced by a broader class of utilities:

$$u_c(Z) = \begin{cases} 
-\infty & \text{if } Z_c \text{ violates one of } c\text{'s quotas} \\
\sum_{(s, c, t) \in Z_c} (1 - P^c_s + \kappa(s, c, t)) & \text{otherwise}
\end{cases}$$

where $\kappa(\cdot, \cdot, \cdot) \geq 0$ is some arbitrary function.

We note that the above-mentioned conditions of Hatfield and Kominers (2015) still hold. Thus, DA terminates in a stable allocation.

Recall that the definition of a regular sequence of markets made no restriction on colleges’ preferences beyond the assumptions on quotas and that all students are acceptable. The following theorem uses the same definition, but allows for colleges’ choice functions that are represented by the above form, as long as the very top students are preferred to the lowest-ranked students, regardless of the contractual terms.

**Theorem 3.** Given a regular sequence of uniform random markets, if $\kappa(s, c, t) < x < 1$ for all $n$ and $(s, c, t) \in S^n \times C^n \times T^n$, then there exists $\Delta > 0$ such that:

1. the expected fraction of students with multiple stable allocations,
2. the expected fraction of colleges with multiple stable allocations that can successfully manipulate DA when all other agents are truthful,
3. the expected fraction of colleges with multiple stable allocations that cannot successfully manipulate DA when all other agents are truthful, and
4. the expected fraction of colleges with different numbers of assigned students in different stable allocations are all greater than $\Delta$. 

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and, if \( x < \frac{1}{2} \), also such that:

5. the expected fraction of students who are matched in some stable allocation, but are unmatched in another, is greater than \( \Delta \).

Proof. That colleges’ choice functions are such that DA terminates in a stable allocation for any profile of students’ preferences follows from Hatfield and Kominers (2015). Thus, the set of stable allocations is not empty.

Consider a college, \( h \), and \( q_{ih}^h + 1 \) students, \( s_1, \ldots, s_{q_{ih}^h + 1} \), all ranked among the highest \( (1 - x)|S^n| \) students on \( h \)’s master list. Endow each of these students, \( s_i \), with artificial preferences of the form \( (s_i, h, l - 1) \succ s_i, (s_i, h, l - 2) \succ s_i \ldots \succ s_i (s_i, h, l_h) \succ s_i \emptyset \). Run DA on a market consisting of \( h \), with \( h \)’s true preferences, and students \( \{s_i\}_{i=1}^{q_{ih}^h + 1} \) with the artificial preferences. Our assumptions assure that one (and only one) of the students will remain unassigned. We denote this student by \( p \) and the others by \( r_1, \ldots, r_{q_{ih}^h} \) consistently with their order on the master list, and consider the event \( \bar{E}^n(p, r_1, \ldots, r_{q_{ih}^h}, h, c) \), which is defined as \( E^n(p, r_1, \ldots, r_{q_{ih}^h}, h, c) \) was previously defined, except that we omit the restriction that \( p \) is ranked on \( h \)’s master list lower than any \( r_i \).

Our selection of \( p \) assures that in the event \( \bar{E}^n(p, r_1, \ldots, r_{q_{ih}^h}, h, c) \), \( p \) is not assigned to \( h \) in the allocation resulting from DA. Furthermore, similarly to the proof of Theorem 2, there is another stable allocation where \( p \) is assigned to \( h \) and one of the \( r \)-students receives a lower level of financial aid. Moreover, \( h \) can manipulate DA and achieve this allocation. The key is that since \( p \) is in the top \( (1 - x) \) percent of \( h \)’s master list, the gain from recruiting him, which is greater than \( x \), more than compensates for any possible loss due to the change in allocation of financial aid, which is bounded above by \( x \).

Finally, our selection of \( p \) was independent of students’ preferences, and therefore students’ preferences are independent of the labels (which depend only on colleges’ preferences). Thus, the lower bound on the probabilities of valid events of the form \( E^n(p, r_1, \ldots, r_{q_{ih}^h}, h, c) \) extends to events of the form \( \bar{E}^n(p, r_1, \ldots, r_{q_{ih}^h}, h, c) \). Furthermore, there are still “many” ways to select \( p, r_1, \ldots, r_{q_{ih}^h} \) : the number shrinks by a factor of \( (1 - x)^{q_{ih}^h + 1} \) but remains of the same order of magnitude.

This completes the proof of the first four parts of the theorem. The proof of the fifth part is analogous, and omitted for brevity. The restriction, in this
part, to the case that $x$ is lower than one half stems from the fact that in our construction (which we used to prove part 5 of Theorem 2) two students assigned to $h$ change their assignment: one receives a lower level of aid still in $h$ and the other one becomes unmatched. Thus, $h$ may lose up to $2x$. 

\[ \square \]

E Market Power with Unattainable Desirable Alternatives

We extend Example 3 to show that a college may possess market power over a student even if the student does not rank the contracts with the college contiguously.

Example 4. We add to Example 3 another (elite) college $e$, and a (genius) student $g$. The elite college has one seat and one level of funding ($q_{e}^0 = 1$, $q_{e}^i = 0$), and its preferences are summarized by

\[ (g, e, 0) \succ (r, e, 0) \succ (p, e, 0) \succ \emptyset. \]

Student $g$’s first-choice college is $e$. Her preferences are given by

\[ (g, e, 0) \succ (g, h, 1) \succ (g, h, 0) \succ (g, c, 1) \succ (g, c, 0) \succ \emptyset. \]

Other students’ preferences are now

\[ (r, h, 1) \succ (r, e, 0) \succ (r, h, 0) \succ (r, c, 1) \succ (r, c, 0) \succ \emptyset, \]

and

\[ (p, h, 1) \succ (p, c, 1) \succ (p, c, 0) \succ (p, h, 0) \succ (p, e, 0) \succ \emptyset. \]

Thus, the elite college and the genius applicant are the first choice of one another, and hence must be matched in any stable allocation. There are two stable allocations under these preferences: \{(r, h, 1), (p, c, 0), (g, e, 0)\}, which is the result of the student-proposing DA, and \{(r, h, 0), (p, h, 1), (g, e, 0)\}.

Had the elite college been interested in the rich applicant (e.g., if its capacity was 2), then the unique stable allocation would be \{(r, h, 1), (p, c, 0), (g, e, 0)\}. Intuitively, the fact that the rich applicant prefers the elite institution to the unfunded position at $h$, combined with the institution’s willingness to accept the rich applicant, eliminates $h$’s ability to apply market power over the rich applicant. That $h$ can apply market power over $r$ is a result of $e$ not being interested in $r$, which means that $r$ does not have an outside option he prefers.
F Computational Experiments

In this appendix, we present simulation results that complement our theoretical and empirical findings. We first present simulation results for the special case that is covered in the body of the paper. Then, we consider a more flexible model in terms of students’ preferences distribution and of colleges’ capacities.

F.1 Uniform Random Markets with Two-Seats One-Scholarship Colleges

We start by concentrating on the special case that was discussed in Section 3. We consider uniform random markets with 10–200 colleges, each with two seats and one available scholarship. For each market size, the number of students is equal to the number of college seats, and students’ preferences are drawn uniformly at random from all acceptable permutation over contracts with 5 colleges (i.e., \( k = 5 \)). College ROLs are also drawn independently and uniformly at random.

For each market size we draw 1000 random markets. We then calculate three statistics. The first statistic is the fraction of colleges that can improve their allocation by applying local market power (i.e., by declaring that the student who is assigned to the college with funding under DA is not eligible for funding).

Second, we calculate a lower bound for the fraction of students who have different stable allocations. This lower bound is calculated by checking, for each of the above-mentioned manipulations, whether the resulting allocation is stable (with respect to the true preferences). We report the fraction of students who receive an allocation other than their DA allocation in at least one of these stable allocations.\footnote{To see that this is a lower bound note, for example, that we ignore potential deviations by groups of colleges.}

Third, we ask how many more students (relative to the DA outcome) can be assigned to some college without compromising stability. We calculate a lower bound for this quantity by comparing the outcome of DA to the largest of two other stable allocations. The first stable allocation is calculated using a greedy algorithm that tries to find a large subset of the colleges identified above who can all apply local market power simultaneously with-

\[ \text{To see that this is a lower bound note, for example, that we ignore potential deviations by groups of colleges.} \]
out compromising stability (with respect to true preferences). The second stable allocation is calculated by flipping the order of funded and unfunded contracts whenever they appear consecutively on an ROL, and running DA. Of note, we do not claim that these are the largest stable allocations, but we are not aware of an efficient way to compute a maximal allocation.

The results are presented in Figure 1. In accordance with the predictions of Theorem 2, all three quantities are bounded away from zero. Note, however, that while the bound we provide in the body of the paper is equal to $6.6 \times 10^{-10}$ under these parameters, the simulated statistics are substantially higher. Panel 1a shows that for all simulated market sizes, the fraction of colleges that can successfully apply local market power when all others are truthful is higher than 20%. Panel 1b shows that the fraction of students with a different assignment in different stable allocations is close to one half. Finally, Panel 1c shows a lower bound of approximately 1.5% for the increase in the number of assigned students by shifting from DA to another stable allocation across market sizes.

F.2 Flexible Preferences Distributions

In this section, we add several parameters to the distribution of preferences and evaluate the robustness of the phenomena we identified to changes in these parameters. Specifically, we allow for a fraction of the population to be “poor,” meaning that these agents only consider funded contracts, and we vary the number of colleges each student considers. For simplicity, “rich” students rank contracts lexicographically in the identity of the college, then financial terms. In what follows, we vary each of these parameters, holding the values of other parameters fixed.

Figure 2 evaluates the effect of changes to $k$, the number of colleges considered by each student. It replicates Figure 1 above, this time with half the population being poor and half rich, for different values of $k$. The findings are qualitatively similar, with the only difference being that with larger values of $k$ the variation in size of stable allocations becomes smaller (as more students are matched under DA to begin with).

Figure 3 presents the results from changing the fraction of poor students. Here, we set the number of colleges to equal 200, and set $k$, the number

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42 We have experimented with other specifications in which the importance of funding varied, and got similar results.
Figure 1: Different measures of the size of the set of stable allocations and the scope for exercising local market power by colleges. Each dot represents the average over 1,000 simulated uniform random markets with two-seats one-scholarship colleges, with the number of colleges corresponding to the number on the horizontal axis. Panel 1a presents the fraction of colleges that can successfully apply local market power when all others are truthful. Panel 1b presents a lower bound for the fraction of students with multiple stable allocations. Panel 1c presents a lower bound for the percentage change in the number of assigned students between the largest stable allocation and the DA outcome.

of colleges each student considers acceptable, to equal 10. Across the three panels, our estimates are maximized in intermediate values of the fraction of poor students. Intuitively, this assures that there are sufficiently many “rich” applicants over which the college can apply market power, and sufficiently many price sensitive students that may be recruited using funding.
Figure 2: Different measures of the size of the set of stable allocations and the scope for exercising local market power by colleges. Each dot represents the average over 1,000 simulated markets with two-seats one-scholarship colleges, where half of the students are poor, and with the number of colleges corresponding to the number on the horizontal axis. Each of the three lines corresponds to a different number of colleges considered by each student. Panel 2a presents the fraction of colleges that can successfully apply local market power when all others are truthful. Panel 2b presents a lower bound for the fraction of students with multiple stable allocations. Panel 2c presents a lower bound for the percentage change in the number of assigned students between the largest stable allocation and the DA outcome.

G Other Models of Large Two-sided Matching Markets

This appendix provides analogues of our results in other models of large two-sided matching markets. For concreteness, we study specific examples where key findings of Azevedo and Leshno (2016) and Azevedo (2014) cease to hold,
Figure 3: Different measures of the size of the set of stable allocations and the scope for exercising local market power by colleges. Each dot represents the average over 1,000 simulated markets with 200 two-seats one-scholarship colleges with the fraction of poor students corresponding to the number on the horizontal axis. Panel 3a presents the fraction of colleges that can successfully apply local market power when all others are truthful. Panel 3b presents a lower bound for the fraction of students with multiple stable allocations. Panel 3c presents a lower bound for the percentage change in the number of assigned students between the largest stable allocation and the DA outcome.

and do not review the general models.

G.1 Multiplicity in Azevedo and Leshno (2016) Markets

Consider a market with two Colleges (\(C = \{a, b\}\)) and a unit mass of students. Each college has two types of seats: funded and unfunded. Each
college has a mass of 1/6 seats of each type.

Students have four payoff types.

- a-rich: $a^f > a^u > b^f > b^u$
- b-rich: $b^f > b^u > a^f > a^u$
- a-poor: $a^f > b^f > a^u > b^u$
- b-poor: $B^f > a^f > b^u > a^u$

Students also have a fit type, $x \in [0, 1]^2$, where $x_1$ is the fit of the student to with college $a$ and $x_2$ is the fit of the student to with college $b$. The mass of students of each payoff type is 1/4, and for each payoff type, fit types are uniformly distributed. College $i$’s utility can be represented by the integral of the fit of the incoming cohort as long as quotas are not exceeded (in which case the college’s utility is $-\infty$).

Figure 4 illustrates the outcome of DA. We calculate it just as Azevedo and Leshno (2016) do. The dark red (blue) students are funded in $b$ ($a$), the pink (light blue) students are unfunded in $b$ ($a$), and the students in white are unassigned.

Of note, program $a$ has market power over all $a$-rich students who are assigned with financial aid, and over all $a$-poor students who are assigned with financial aid and do not meet the bar for financial aid in $b$ ($\approx .8$ $b$-fit).

Also note that the least desirable students assigned to $a$ (those closest to the white rectangle in the light blue rectangle) are less desirable than the most desirable $a$-poor students not assigned to $a$ (those closest to the dark blue rectangle who are in the dark red rectangle). These students would rather attend $a$ if they are offered funding.

Our main observation is that by refusing to fund some students over whom the college has market power, the college $a$ can “poach” students from the latter group. In this example, since the college $a$ has no free capacity, it will also need to give up on some of the less desirable (unfunded) students.

Figure 5 is an illustration of the stable allocation resulting from both colleges applying such strategy. This case is relatively simple, as symmetry guarantees that the number of “poached” students (who are all funded) is equal to the number of students “poached” by the other school, freeing up exactly the required number of seats and scholarships. Of note, symmetry only simplifies the calculations, but our construction clearly generalizes to other environments.
Figure 4: The stable allocation resulting from DA. Each square represents \(\frac{1}{4}\) mass of students with the corresponding preference profile. The \(x\) (\(y\)) coordinate represents the student’s fit in college \(a\) (\(b\)). Dark blue (red) represents assignment to \(a\) (\(b\)) with funding. Light blue (pink) represents assignment to \(a\) (\(b\)) without funding. Students in the white regions are unassigned.
Figure 5: A stable allocation where both colleges use need based financial aid. Each square represents \( \frac{1}{4} \) mass of students with the corresponding preference profile. The \( x \) (\( y \)) coordinate represents the student’s fit in college \( a \) (\( b \)). Dark blue (red) represents assignment to \( a \) (\( b \)) with funding. Light blue (pink) represents assignment to \( a \) (\( b \)) without funding. Students in the white regions are unassigned.
Note that an \(a\)-poor applicant with fit \(0.7(\in [\frac{1}{\sqrt{3}}, 0.8])\) in \(a\) and fit of 1 in \(b\) is funded in \(a\) in this stable allocation, but a student who is identical except that his fit in \(b\) is 0 is assigned to \(a\) without funding. The reason is that \(b\)-funded is a feasible outside option only for the former student. Similarly, an \(a\)-rich applicant with fit \(0.7(\in [\frac{1}{\sqrt{3}}, 0.8])\) in \(a\) and fit of 1 in \(b\) is assigned to \(a\) without funding in this stable allocation. The reason for the difference is the absence of an outside option due to the strength of his preferences. Local market power for colleges is absent only when preferences for the college are not strong (there exists a desired alternative) \textbf{and} this alternative is feasible.

Finally, we identify a third stable allocation. We think of this case as college \(b\) using need based financial aid and college \(a\) sticking to the merit based approach. The allocation is depicted in Figure 6.

In the stable allocation depicted in this figure admission to college \(b\) becomes more selective and admission to college \(a\) becomes less selective (under any terms). Moreover, college \(b\) derives higher utility relative to DA, and college \(a\) derives lower utility. With sufficient information, this allocation can be achieved by \(b\) manipulating DA, while all others being truthful. The white area (unassigned students) does not fully overlap the white area in the other stable allocations depicted above, illustrating a violation of the rural hospital theorem in this large market.

\section*{G.2 Manipulation by Small Colleges in an \textcolor{green}{Azevedo (2014)} Market}

We now turn to a generalized version of the example above. In this market, a large part of the population is assigned to different colleges in different stable allocations and colleges, although “small” in the sense of \textcolor{green}{Azevedo (2014)}, can successfully manipulate DA. This example is also interesting since the probability that an agent ranks two contracts with the same college consecutively approaches 0.

There are \(n\) colleges, each with \(\frac{1}{3n}\) funded seats and \(\frac{1}{3n}\) unfunded seats. This preserves the mass of available seats throughout, but each college’s size is vanishing as \(n\) grows large. Students’ preferences are distributed uniformly over all complete acceptable rank orders (i.e., any allocation is better than no allocation, and funding in the same program is always preferred). Students’ fit with programs is uniformly distributed on the unit hyper-cube, independently of their utility type.
Figure 6: A stable allocation where only college b uses need based financial aid. Each square represents $\frac{1}{4}$ mass of students with the corresponding preference profile. The $x$ ($y$) coordinate represents the student’s fit in college $a$ ($b$). Dark blue (red) represents assignment to $a$ ($b$) with funding. Light blue (pink) represents assignment to $a$ ($b$) without funding. Students in the white regions are unassigned.
First, note that the argument from the 2 college case extends to this setting, so the stable allocation corresponding to the outcome of DA can be represented by two cutoffs, $x_n$ (for unfunded seats) and $y_n$ (for funded seats), due to the symmetry of the market. Furthermore, since $\frac{1}{3}$ students must be unassigned due to capacity constraints, and since any allocation is individually rational for students, it must be that $(x_n)^n = \frac{1}{3}$. Hence, $(x_n) = (\frac{1}{3})^n$.

Next, note that any student who is assigned to some college with funding, does not pass any cutoff of any other college with probability greater than $\frac{1}{3}$. To see this, note that $\frac{1}{3}$ is the ex-ante probability that the student does not meet any cutoff, which is lower than the probability of not meeting any cutoffs of $n - 1$ colleges, and that unless the student is assigned to his first choice, this probability should be updated upwards (as we learn that the student did not meet certain cutoffs).

A corollary of the above is that in the allocation resulting from DA, each college has market power over at least a third of its funded students. Since a third of the students are funded in this allocation, this implies that at least $\frac{1}{9}$ of the students receive funding even though the college can refuse to fund them without compromising stability. We now show that, as in the case with 2 colleges, there are sufficiently many desirable students whose choices are distorted due to funding. That is, by reallocating some scholarships, the college can “poach” good fit students from other colleges.

**Lemma 4.** There exists $C > 0$ independent of $n$, such that for all $n > 1$ there exists a stable allocation in which each college has at least a fraction $C$ of its student body different than the outcome of DA.

**Proof.** Consider the set of students $E_n[a, b]$ such that:

1. each student in the set has fit lower than $x_n$ in all programs other than $a$ and $b$.

2. each student ranks $(s, a, 1)$ over $(s, b, 1)$ over $(s, b, 0)$ over $(s, a, 0)$. These contracts do not have to be ranked consecutively.

3. each student has fit for $a$ in the interval $(\frac{x_n + y_n}{2}, y_n)$. In other words, the students’ fit with $a$ is like the “high end” of unfunded students’ in $a$, but they do not meet the bar for funding under DA.
4. each student has fit for \( b \) in the interval \((y_n, 1)\). In other words, the student is eligible for funding in \( b \) in the allocation resulting from DA.

First, note that students in \( E_n[a, b] \) are assigned to \( b \) with funding under DA, but they would rather attend \( a \) with funding. Second, \( a \) is happy to reject the lower ranked unfunded students, apply market power on some funded students, and “poach” students in \( E_n[a, b] \). In fact, in this example, this will not be necessary: Since the sets \( E_n[a, b] \) are disjoint for any ordered selection of a pair of colleges \( a \neq b \), we can “swap” these sets between colleges so that in the resulting allocation “swapped” students are funded (and take exactly the funded seats of “poached” students, by symmetry).

It remains to show that the mass of the sets \( E_n[a, b] \) is greater than \( \frac{c}{n^2} \) for some \( c > 0 \). This will suffice since there are \( n(n-1) \) ways to choose ordered pairs of colleges, and the sets \( E_n[a, b] \) are disjoint for different selections. This holds since all conditions are independent, and so the probability of the event is the product of the probabilities of each condition. Furthermore, the probability of condition 1 is greater than \( \frac{1}{3} \) as discussed above, the probability of condition 2 is \( \frac{1}{2} \), the probability of condition 3 is \( \frac{y_n-x_n}{2} \), and the probability of condition 4 is \( 1 - y_n \).

Since all funded positions are filled under DA, \( 1 - y_n \geq \frac{1}{3n} \) (the set of students eligible for funding is at least as large as the set of students who choose to go to the college with funding, i.e., do not have a preferred outside option). Furthermore, since fewer people compete for unfunded positions (or by direct calculation) \( y_n - x_n \geq 1 - y_n \). Thus, \( \frac{y_n-x_n}{2} \geq \frac{1}{6n} \). Hence \( \Pr\{\cup E_n[a, b]\} > n(n-1)\frac{c}{n^2} > \frac{c}{2} > 0 \).

\[ \square \]

**Remark 1.** Our choice of the interval \((\frac{x_n+y_n}{2}, y_n)\) was conservative. We could have chosen \((x_n, y_n)\). We made this choice to highlight how a single college can manipulate DA when others are truthful (without running the risk that \( "x_n" \) will rise too high as to make it regret some poached students). This argument is a straightforward generalization of the last part of the previous section.

**H Evidence from the IPMM**

We complement the analysis from Section 4 by studying data from the Israeli Psychology Master’s Match. We provide evidence in support of our
assumptions on the demand structure, and corroborate the predictions of our model.

Background

In Israel, admissions to graduate programs in psychology are highly selective. Each year, about 1,400 students graduate from a bachelor’s program, but this does not certify them to serve as therapists. In order to become a therapist, one needs to complete a clinical graduate degree and later an apprenticeship. But seats in clinical programs are scarce: only 300 students are accepted each year and about 300 students are accepted to other, non-clinical, programs.

While the number of applicants – approximately 1,000 a year – far exceeds the number of available seats, departments of psychology still compete for top talent. In an attempt to attract “star” applicants, several departments offer a limited number of prestigious scholarships to selected students. In 2014, when this market was centralized, it was critical to allow these departments to continue to pursue this recruiting strategy. Thus, the version of DA that is used in this market allows programs to offer contracts with multiple funding levels, and allows applicants to rank these alternatives separately.43

Data

We use the dataset prepared by Hassidim, Romm and Shorrer (2016). The data includes administrative match data, including ROLs and program reports from the 2014 and 2015 matches. Additionally, it includes the computer code for the market-clearing algorithm.

In 2014, 10 programs in 3 departments offered admission under multiple funding levels. The number increased to 15 programs in 4 departments in 2015. Funding levels ranged from approximately $2,000 a year to approximately $20,000 a year. The number of available scholarships was 25 in 2014, and 36 in 2015.

Each year about 1,000 students participated in the match. The number of ROLs ranking some contract with a program offering multiple funding levels was 271 in 2014, and grew to 458 in 2015, as a result of the growth in the number of programs offering admission with multiple levels of funding.

43The mechanism used by the Israeli Psychology Master’s Match is strategy-proof for students. For more details about the market and the mechanism, see Hassidim, Romm and Shorrer (2017).

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(i.e., if we ignore observations attributed to new programs only, the number remained almost constant).

**Student rank-order lists**

Participants ranked, on average, 4.32 contracts ($\sigma = 4.14$). About 37.2% of the participants ranked at least one of the programs that offered admission under multiple financial-aid levels. Of these, only 3.4% ranked only the funded contract in some program, but not the unfunded contract. Among the applicants who ranked both a funded and an unfunded contract, more than 90% ranked the funded contract first. Among these applicants, the mean number of contracts ranked between a funded contract and the unfunded contract with the same program was 0.34. In 82.6% of the cases, the number was zero.

**Stable allocations and manipulations**

While we have access to all match data, there are still data limitations that do not allow us to calculate the set of stable allocations fully. The main issue is that only the parts of the departments’ preferences that were required to calculate the outcome of DA were elicited. And departments, which typically offer several study programs, often have complex preferences. An additional issue is that we are not aware of an efficient way to calculate the set of stable allocations.

Instead, we take an approach similar to the one we used in the Hungarian dataset for detecting applicants over whom the program may be able to apply market power ($MP_c$). Next, we declare them as ineligible for funding in that program (by changing the department’s preference report) and rerun the match. Finally, we check if the match is stable with respect to true preferences (i.e., if the applicant and the department are part of a blocking coalition of the resulting match). To do so, we require the assumption that programs do not care directly about the identities of the recipients of financial aid, but only about the quality of the incoming cohort. Based on our discussions with department chairs and recruiting committees during the design of the centralized clearinghouse, we are very comfortable with this assumption.

We find that, with a few exceptions, programs have market power over the recipients of financial aid. And while they can reallocate funding among admitted students, applying market power will not improve the quality of
their incoming cohort. Our findings are reminiscent of Claim 1 in Example 1, which is not surprising in light of the fact that in 82.6% of the cases the funded and unfunded contracts in the same program were ranked consecutively.\footnote{In the 2016 match, two out of the four departments that had offered multiple levels of funding decided to offer identical terms to all students admitted to the same program.}