School choice

Market Design Mini-Course, HUJI
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What's in today's class?

- Case studies: NYC and Boston
- Algorithms: DA, Boston mechanism, TTC
- More on tie-breaking

What is different about school choice?

- Schools and students as strategic players
- True indifferences
- The concept of justified envy
- Stability vs. optimality

The NYC High School Match

- Until 2002:
  - Decentralized applications and admissions
  - Only five choices allowed
  - Three rounds of waiting lists, waiting lists run by mail
  - Congestion (out of over 90,000 kids every year, 30,000 administratively assigned, and 17,000 receiving multiple offers)
  - 30,000 students assigned to schools not on their choice list
  - Schools take students' ranking into account
  - Withholding of capacity

The NYC High School Match

- Are NYC schools really two-sided matching problem?
  - Schools conceal capacities
  - EdOpt schools have different preferences (high scores, attendance records, etc.)

- Solution: Deferred Acceptance
  - Only 12 options allowed (breaks truthful revelation, Haeringer and Klijn, JET, 2009)
  - Due to historical rules about specialized schools – matching is done in 3 rounds (round 3 for unmatched kids)

The NYC High School Match

- Single tie-breaking vs. multiple tie-breaking (for schools' indifferences)?
  - Multiple tie-breaking increases number of instabilities, and it therefore constrains the efficiency
  - NYC DOE were presented with simulations, and tried both tie-breaking rules, and decided on single tie-breaking rule

The NYC High School Match

- Outcome:
  - Only 3,000 did not receive any school they chose (compare to 30,000 the previous year)
  - The reasons: relieving congestion (many offers and acceptances, instead of only three rounds), giving each student a single offer (instead of people getting multiple offers), allowing ranking of 12 instead of 5 schools, but also…
  - The results continued to be better and better each year (comparing rankings), even though there were no changes to the algorithm… hmmm…
  - The answer: schools have learned to stop withholding capacity!
  - Open question: how to do appeals? (TTC? but that's later)

Boston Public Schools (BPS)

- About 4000 kids in each cohort. Four cohorts are making choices: K, 1, 6, and 9.
- Priorities (= schools' preferences) come from walking zones, siblings, and random tie-breaking

Boston Public Schools (BPS)

- Until 2006 the mechanism used is "The Boston Mechanism" (but also used in many other places):
  - Step $k.1$: Each student that is still unmatched applies to her most preferred school
  - Step $k.2$: Each school fills its quota as much as possible with those applicants that it prefers the most, and rejects the rest

- Problems with the "Boston Mechanism":
  1. Does not produce stable matchings
  2. Truth-telling is not dominant (far from it)
  3. Not immediately clear that something is wrong…
  4. Those who do not play strategically get hurt

Boston Public Schools (BPS)

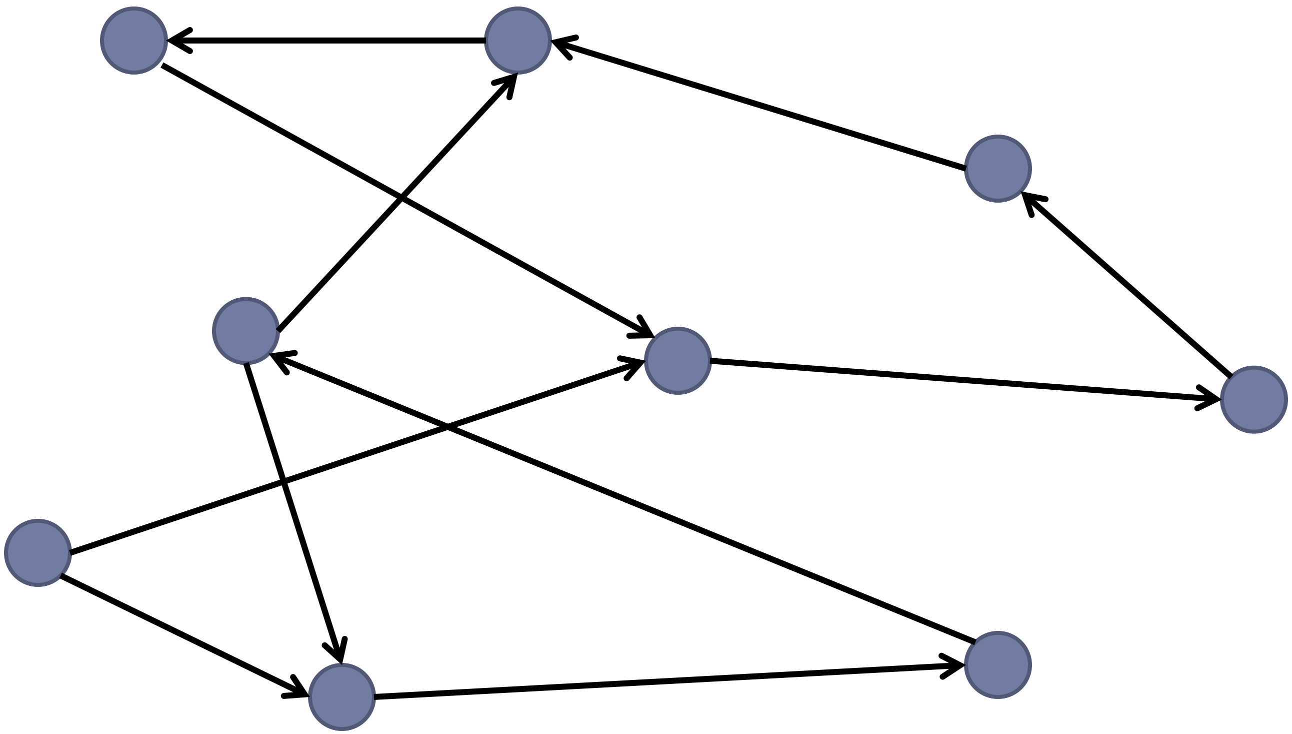
- Unlike NYC – unclear that the market is two-sided:
  - No gaming by schools
  - Lots of people in each priority class, and looks like priorities are meant to help parents select schools
  - If the market is actually one-sided, then stable matchings are not Pareto optimal (it is better for people to trade priorities)

Top Trading Cycles (TTC)

- Introduced in Shapley and Scarf (1974), but attributed to David Gale.

- Draw a graph where each agent is a node, with each agent pointing to his/her/its most preferred match.
- Remove a cycle, and redraw the edges, now each agent points to most preferred match among those remaining.
- Repeat until all nodes are removed.

Top Trading Cycles



Top Trading Cycles

Theorem (Shapley and Scarf, 1974): the outcome of TTC is in the core.

Theorem (Roth, 1982): TTC is strategyproof.

Boston Public Schools (BPS)

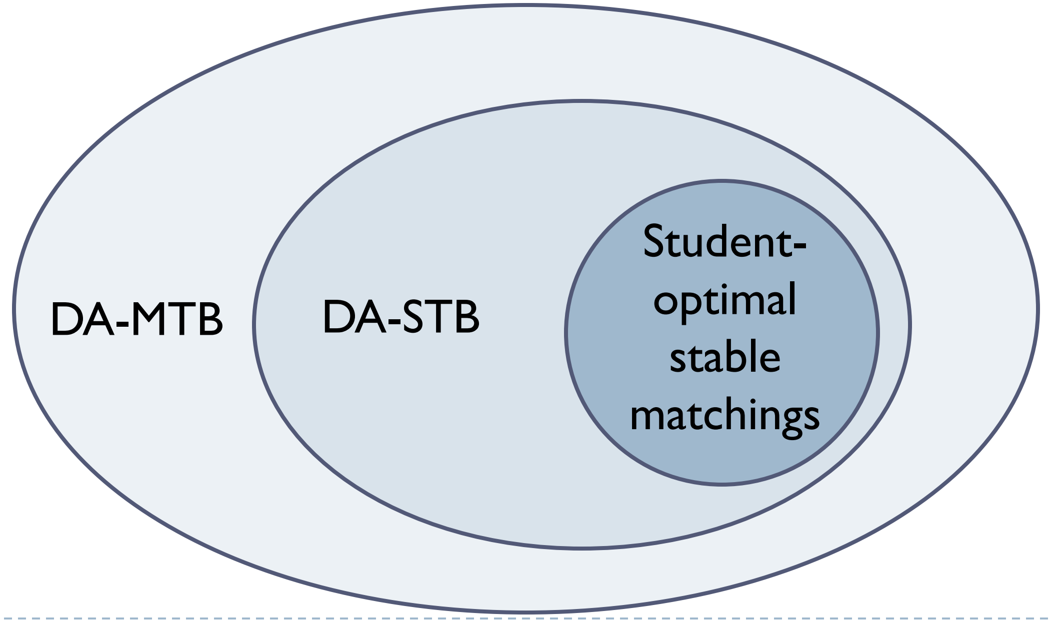
- So there were two options for Boston:
  - DA – Strategyproof, stable, selects student-optimal matching (except for tie-breaking issues)
  - TTC – Strategyproof, Pareto efficient for the students

- The most important thing: that the algorithm will be strategyproof. This levels the playing field and allows gathering data about actual preferences over schools.

- The DA algorithm was chosen because it is more transparent and easier to explain to parents.

A bit more on tie-breaking

Proposition: For any set of strict preferences for students and weak preferences for schools, any matching that can be produced by deferred acceptance with multiple tie-breaking, but not by deferred acceptance with single tie-breaking is not a student-optimal stable matching.



A bit more on tie-breaking

- Example:

There are three schools $S = \{s_1, s_2, s_3\}$ and three students $I = \{i_1, i_2, i_3\}$.

$s_2 \succ_{i_1} s_1 \succ_{i_1} s_3$ $\quad\quad$ $i_1 \sim_{s_1} i_2 \sim_{s_1} i_3$

$s_1 \succ_{i_2} s_2 \succ_{i_2} s_3$ $\quad\quad$ $i_2 \succ_{s_2} i_1 \succ_{s_2} i_3$

$s_1 \succ_{i_3} s_2 \succ_{i_3} s_3$ $\quad\quad$ $i_3 \succ_{s_3} i_1 \succ_{s_3} i_2$

Three stable matchings from student-proposing DA with different tie-breaking rules:

$$\mu_1 = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix} \quad \mu_2 = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_2 & s_1 & s_3 \end{pmatrix} \quad \mu_3 = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_3 & s_2 & s_1 \end{pmatrix}$$

Note that while all are stable, μ_1 is not student-optimal, because μ_2 dominates μ_1.

Stable improvement cycles

- Based on Erdil and Ergin (AER, 2008)
- Given a stable matching μ, strict preferences for students and priorities for the schools, a **stable improvement cycle** consists of students $i_1, \dots, i_n = i_0$ such that:
  1. $\mu(i_k) \in S$ (every student is matched to a school)
  2. $\mu(i_{k+1}) \succ_{i_k} \mu(i_k)$ (every student prefers the school the next student is currently allocated)
  3. $i_k \in \arg\max\{ j \mid \mu(i_{k+1}) \succ_j \mu(j) \}$, where the argmax is taken with respect to school $\mu(i_{k+1})$'s priorities.

- Given a stable improvement cycle create a new matching:

$$\mu'(j) = \begin{cases} \mu(j) & j \notin \{i_1, \dots, i_n\} \\ \mu(i_{k+1}) & j = i_k \end{cases}$$

<u>Proposition</u>: μ' is stable and it (weakly) Pareto dominates μ.

Stable improvement cycles

Theorem: Fix the preferences and priorities, and let μ be a stable matching. If μ is (weakly) Pareto dominated by another stable matching, then μ admits a stable improvement cycle.

Corollary: In order to find a student-optimal stable matching, we can run deferred acceptance, and then find and implement stable improvement cycles until none are left.

Stable improvement cycles

Table 1— Tie-breaking for Grade 8 Applicants in NYC in 2006-07

Choice	Deferred Acceptance Single Tie-Breaking DA-STB (1)	Deferred Acceptance Multiple Tie-Breaking DA-MTB (2)	Student-Optimal Stable Matching (3)	Improvement from DA-STB to Student-Optimal	Number of Students (4)
1	32,105.3 (62.2)	29,849.9 (67.7)	32,701.5 (58.4)	+1	633.2 (32.1)
2	14,296.0 (53.2)	14,562.3 (59.0)	14,382.6 (50.9)	+2	338.6 (22.0)
3	9,279.4 (47.4)	9,859.7 (52.5)	9,208.6 (46.0)	+3	198.3 (15.5)
4	6,112.8 (43.5)	6,653.3 (47.5)	5,999.8 (41.4)	+4	125.6 (11.0)
5	3,988.2 (34.4)	4,386.8 (39.4)	3,883.4 (33.8)	+5	79.4 (8.9)
6	2,628.8 (29.6)	2,910.1 (33.5)	2,519.5 (28.4)	+6	51.7 (6.9)
7	1,732.7 (26.0)	1,919.1 (28.0)	1,654.6 (24.1)	+7	26.9 (5.1)
8	1,099.1 (23.3)	1,212.2 (26.8)	1,034.8 (22.1)	+8	17.0 (4.1)
9	761.9 (17.8)	817.1 (21.7)	716.7 (17.4)	+9	10.2 (3.1)
10	526.4 (15.4)	548.4 (19.4)	485.6 (15.1)	+10	4.7 (2.0)
11	348.0 (13.2)	353.2 (12.8)	316.3 (12.3)	+11	2.0 (1.1)
12	236.0 (10.9)	229.3 (10.5)	211.2 (10.4)		
unassigned	5,613.4 (26.5)	5,426.7 (21.4)	5,613.4 (26.5)	Total:	1,487.5

Stable improvement cycles

Theorem (Abdulkadiroglu, Pathak and Roth, 2008): For any tie breaking rule τ, there is no mechanism that is strategy-proof and dominates DA^{τ}.

Furthermore, when considering stable improvement cycles, it is kind of clear what kind of manipulations might be profitable. It is worthwhile to list schools that are over-demanded and in which you might have priority in order to replace them with people who have priority in other schools that you actually want.

Stable improvement cycles

<u>Example</u> (Azavedo and Leshno, 2010):

$I = \{i_1, i_2, i_3, i_4\}, S = \{s_1, s_2\}$ with $q_{s_1} = 1$ and $q_{s_2} = 2$.

$s_2 \succ_{i_1} s_1 \succ_{i_1} \emptyset$ $\quad$ $i_1 \succ_{s_1} i_2 \sim_{s_1} i_3 \sim_{s_1} i_4$

$s_2 \succ_{i_2} \emptyset$ $\quad$ $i_3 \sim_{s_2} i_4 \succ_{s_2} i_1 \sim_{s_2} i_2$

$s_1 \succ_{i_3} \emptyset$

$s_1 \succ_{i_4} \emptyset$

Assume utility from first choice is 1, from staying single is 0, and that $u_{i_3}(s_2) > -\frac{1}{2}$ and $u_{i_4}(s_2) > -\frac{1}{2}$.

With DA-STB with random tie-breaking the equilibrium is truthful revelation, and allocation is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ s_2 & s_2 & \frac{1}{2}s_1 + \frac{1}{2}\emptyset & \frac{1}{2}s_1 + \frac{1}{2}\emptyset \end{pmatrix}$$

Stable improvement cycles

If, however, both i_3 and i_4 report the preference $s_1 \succ s_2 \succ \emptyset$ then the DA-STB allocation is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ \frac{1}{2}s_1 + \frac{1}{2}s_2 & \emptyset & \frac{1}{4}s_1 + \frac{3}{4}s_2 & \frac{1}{4}s_1 + \frac{3}{4}s_2 \end{pmatrix}$$

and the unique Pareto efficient assignment (with respect to reported preferences) that dominates DA-STB is

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ s_2 & \emptyset & \frac{1}{2}s_1 + \frac{1}{2}s_2 & \frac{1}{2}s_1 + \frac{1}{2}s_2 \end{pmatrix}$$

And this is equilibrium.

<u>Corollary</u>: Consider any mechanism that is Pareto efficient with respect to reported preferences, and Pareto dominates DA-STB. In the economy described, this mechanism has a unique equilibrium assignment which is Pareto dominated by the DA-STB assignment, and is unstable with respect to the true preferences.

What's in the next class?

- Signaling
- Object allocation
  - Algorithms: Random Serial Dictatorship, Probabilistic Serial, Linear Programming (for rank-order dominating assignments)
  - Some large market results